International Friends and Enemies*

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Abstract

We develop sufficient statistics of countries' bilateral income and welfare exposure to foreign productivity shocks that are exact for small shocks in the class of models with a constant trade elasticity. For large shocks, we characterize the quality of the approximation, and show it to be almost exact. We compute these sufficient statistics for over 140 countries from 1970-2012. We show that our exposure measures depend on market-size, cross-substitution and cost of living effects. As countries become greater economic friends in terms of welfare exposure, they become greater political friends in terms of United Nations voting and strategic rivalries.

Keywords: productivity growth, trade, welfare

JEL Classification: F14, F15, F50

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1 Introduction

One of the most dramatic changes in the world economy over the past half century has been the emergence of China as a major force in world trade. A central question in international economics is the implications of such economic growth for the income and welfare of trade partners. A related question in political economy is the extent to which these large-scale changes in relative economic size necessarily involve heightened political tension and realignments in the international balance of power. We provide new theory and evidence on both of these questions by developing bilateral "friends" and "enemies" measures of countries' income and welfare exposure to foreign productivity shocks that can be computed using only observed trade data. Our measures are derived directly from the leading class of international trade models with a constant trade elasticity, are computationally trivial to compute, and have a clear economic interpretation even in rich quantitative settings with many countries. We show that our analysis admits a large number of generalizations, including multiple sectors, input-output linkages and factor mobility (economic geography). We derive bounds for the sensitivity of countries' exposure to foreign productivity shocks to departures from a constant trade elasticity. Using standard matrix inversion techniques, we compute over 1 million bilateral comparative statics for income and welfare exposure to foreign productivity growth for more than 140 countries from 1970-2012 in a few seconds. Consistent with the idea that conflicting economic interests can spawn political discord, we find that as countries become greater economic friends in terms of the welfare effects of their productivity growth, they also become greater political friends in terms of the similarity of their foreign policy stances, as measured by United Nations voting patterns and strategic rivalries.

Our research contributes to the recent revolution in international trade of the development of quantitative trade models following Eaton and Kortum (2002) and Arkolakis, Costinot and Rodriguez-Clare (2012). A key advantage of these quantitative models is that they are rich enough to capture first-order features of the data, such as a gravity equation for bilateral trade, and yet remain sufficiently tractable as to be amenable to counterfactual analysis, with a small number of structural parameters. A key challenge is that these models are highly non-linear, which can make it difficult to understand the economic explanations for quantitative findings for particular countries or industries. A key contribution of our bilateral friends-and-enemies measures is to reveal the role played by different economic mechanisms in these models. In particular, we show that the effect of a productivity shock in a given country on welfare in each country depends on three matrices of observed trade shares: (i) an *expenditure share* matrix ($\mathbf{S}$) that reflects the expenditure share of each importer on each exporter; (ii) an *income share* matrix ($\mathbf{T}$) that captures the share of each exporter's value added derived from each importer; (ii) a *cross-substitution* matrix ($\mathbf{M}$) that summarizes how an increase in the competitiveness of one country leads consumers to substitute away from all other countries in each market. Using these results, we separate out countries' welfare exposure to foreign productivity shocks into income and cost-of-living effects; break out income exposure to these productivity shocks into market-size and cross-substitution effects; isolate partial and general equilibrium effects; evaluate the contributions of individual sectors to aggregates; and assess the contribution of importer, exporter and third-market effects.

Our bilateral friends-and-enemies measures are exact for small productivity shocks for this leading class of international trade models characterized by a constant trade elasticity. For large productivity shocks, we provide two sets of analytical results for the quality of our approximation. First, we compare our linearization to the non-linear exact-hat algebra approach that is commonly used for counterfactuals in constant elasticity trade models. We show that the quality of our approximation depends on the properties of the observed trade matrices ($\mathbf{S}$, $\mathbf{T}$ and $\mathbf{M}$). Given the observed values of these matrices and productivity shocks of the magnitude implied by the observed trade data, we find that the two sets of predictions are almost visibly indistinguishable from one another, with a regression slope and R-squared close to one. Second, we compare our results for a constant trade elasticity with those for a variable trade elasticity, and derive sensitivity bounds for the impact of productivity shocks in this more general specification with a variable trade elasticity. As such, our characterization of the incidence of productivity and trade shocks in terms of the market-size, cross-substitution and cost-of-living effects provides a useful benchmark for interpreting the results of quantitative trade models outside of our class.

Our main empirical contribution is to use our friends-and-enemies exposure measures to examine the global incidence of productivity growth in each country on income and welfare in more than 140 countries over more than forty years from 1970-2012. We find a substantial and statistically significant increase in both the mean and dispersion of welfare exposure to foreign productivity shocks over our sample period, consistent with increasing globalization enhancing countries' economic dependence on one another. We find that productivity growth in most countries raises their own income compared to world GDP and reduces the income of most (but not all) other countries compared to world GDP. Even compared to a weighted average of OECD countries, we find that Chinese productivity growth has an increasingly large negative effect on US relative income. Nevertheless, once changes in the cost of living are taken into account, this Chinese productivity growth has an increasingly large positive effect on aggregate US welfare. More generally, we provide evidence of large-scale changes in bilateral patterns of welfare exposure to foreign productivity growth following trade liberalization in North America, the fall of the Iron Curtain in Europe, and the replacement of Japan by China at the center of geographic production networks in Asia.

We decompose overall income and welfare exposure into the direct (partial equilibrium) effect of foreign productivity at initial incomes and the indirect (general equilibrium) effect through endogenous changes in incomes. We find that these general equilibrium forces are quantitatively large in this class of models, such that misleading conclusions about the income and welfare effects of productivity growth can be drawn from simply looking at the partial equilibrium terms alone. We find that both the cross-substitution effect and the market-size effect make substantial contributions to overall income exposure. On the one hand, the partial equilibrium component of the cross-substitution effect is always negative, because higher productivity growth in a given country increases its price competitiveness and leads to substitution away from all other countries. On the other hand, the general equilibrium components of the cross-substitution and market-size effects can be either positive or negative, because higher productivity in a given country raises its own income and affects income in all other countries, which in turn induces changes in price competitiveness and market demand for all countries. Finally, although much of the effect of foreign productivity growth on home income occurs through the home country's market, we also find a substantial effect through the foreign country's market, and empirically-relevant effects through the home country's most important third markets.

We compare our friends-and-enemies exposure measures in our baseline model with a single sector to those in models with multiple sectors and input-output linkages. Although there is a strong correlation between the predictions of all three models, we find that introducing both sectoral comparative advantage and production networks has quantitatively relevant effects on bilateral income and welfare exposure for individual pairs of exporters and importers. Additionally, both the multiple-sector and input-output models yield additional disaggregated sector-level predictions, in which even foreign productivity growth that is common across sectors can have heterogeneous effects across individual industries in trade partners, depending on the extent to which countries compete with one another in sectoral output markets versus source intermediate inputs from one another. Comparing these sector-level predictions for the impact of Chinese productivity growth, we find some marked differences across countries. For nearby South-East Asian countries, the sectors that benefit most include the Electrical, Medical and Office sectors, consistent with input-output linkages between related sectors through global value chains in Factory Asia. However, for the resource-rich emerging economies, the sectors that benefit most include the Mining, Agricultural and Basic Metals sectors, consistent with a form of "Dutch Disease," in which the growth of resource-intensive sectors propelled by Chinese demand competes away factors of production from less resource-intensive sectors.

We use our friends-and-enemies exposure measures to provide new evidence on a political economy debate about the extent to which increased economic rivalry between nations necessarily involves heightened political tension. A number of scholars have drawn parallels between the current China-US tensions and earlier historical episodes, such as the confrontation between Germany and Great Britain around the turn of the twentieth century, and the rise of Athens that instilled fear in Sparta that itself made war more likely (the Thucydides Trap).[1] On the one hand, there are good reasons to be skeptical about this essentially mercantilist view of the world, because a key insight from trade theory is that trade between countries is not zero-sum. On the other hand, it remains possible that the extent to which countries have shared economic interests is predictive of their political alignment. Consistent with this view, we find that as countries become less economically friendly in terms of the welfare effects of their productivity growth, they also become less politically friendly in terms of their foreign policy stances, as measured by United Nations voting patterns and strategic rivalries.

Our research is related to several strands of existing work. First, traditional neoclassical theories of trade highlight the terms of trade as the central economic mechanism through which shocks to productivity, transport costs and trade policies affect welfare in other countries. Key insights from this literature are that foreign productivity growth can either raise or reduce domestic welfare depending on whether it is export or import-biased (e.g. Hicks 1953, Johnson 1955 and Krugman 1994), and immiserizing growth becomes a theoretical possibility if domestic productivity growth induces a sufficiently large deterioration in the terms of trade (see Bhagwati 1958). While this theoretical literature isolates key economic mechanisms, the empirical magnitude of these effects remains unclear, because these theoretical results are typically derived in stylized settings with homogeneous goods and a small number of countries and goods (typically two countries and two goods).

Second, we contribute to the growing literature on quantitative trade models following the seminal and Frisch-medal winning research of Eaton and Kortum (2002), including Dekle et al. (2007), Costinot et al. (2012), Caliendo and Parro (2015), Adão et al. (2017), Burstein and Vogel (2017), Caliendo et al. (2018), and Levchenko and Zhang (2016). Using a multi-sector quantitative trade model, Hsieh and Ossa (2016) find small spillover effects of Chinese productivity growth from 1995-2007 on other countries' welfare, which range from -0.2 percent to 0.2 percent. In a counterfactual analysis of alternative patterns of Chinese productivity growth, di Giovanni et al. (2014) find that most countries experience larger welfare gains when China's productivity growth is biased towards comparative disadvantage sectors. In a specification incorporating many local labor markets within the United States, Caliendo et al. (2019) develop a quantitative trade model that replicates reduced-form empirical findings for the China shock,

[1] See for example Brunnermeier et al. (2018) and "China-US rivalry and threats to globalisation recall ominous past, " Martin Wolf, *Financial Times*, 26th May, 2020.

with net welfare gains for the United States as a whole, but heterogeneity across local labor markets. We contribute to this research by developing new friends and enemies measures of exposure to foreign productivity growth that closely replicate the full nonlinear solution of the leading class of quantitative trade models, while also revealing the role of the key economic mechanisms through which productivity growth in one country affects welfare in another in these models. The low computational burden of our approach lends itself to applications in which large numbers of counterfactuals must be undertaken, as in our empirical application with more than 1 million comparative statics. Furthermore, the wide range of extensions and generalizations of our approach allow researchers to easily compare and contrast the results of large numbers of counterfactuals across different quantitative models, such as our single-sector, multi-sector and input-output specifications.

Third, our work is related to the burgeoning literature on sufficient statistics for welfare in international trade, including Arkolakis et al. (2012), Caliendo et al. (2017), Baqaee and Farhi (2019), Galle et al. (2018), Huo et al. (2019), Bartelme et al. (2019), and Kim and Vogel (2020). Within a class of leading international trade models, Arkolakis et al. (2012) shows that the welfare gains from trade can be measured using only a country's domestic trade share and a constant trade elasticity. In a wider class of gravity models, Allen et al. (2020) use the network structure of trade to prove existence and uniqueness, and show that counterfactual predictions in this class of models have a series expansion representation in terms of demand and supply matrices that are functions of trade data and demand and supply elasticities. In a model with general spatial links between local labor markets, Adão et al. (2019) characterize general equilibrium elasticities of employment, wages, and real wages in each market with respect to shift-share measures of exposure to foreign trade shocks using revenue and consumption shares. In a general network economy, Baqaee and Farhi (2019) derive first-order counterfactual formulas (and second-order accounting formulas) for productivity and trade shocks, and implement these for a nested CES economy. Our main contribution relative to this body of research on sufficient statistics is to derive bilateral friends-and-enemies measures of exposure to foreign productivity shocks that can be directly connected to underlying market-size, cross-substitution and cost of living effects in a large class of quantitative trade models. We show that these friends-and-enemies measures can be recovered from observed income and expenditure share data using standard matrix inversion techniques. Although our measures capture first-order general equilibrium counterfactuals, we provide an analytical characterization of the magnitude of the second and higher-order terms, and hence the quality of our approximation to the full non-linear model solution.

Fourth, our research connects with the large reduced-form literature that has examined the domestic effects of trade shocks (such as the China shock), including Topalova (2010), Kovak (2013), Dix-Carneiro and Kovak (2015), Autor et al. (2013), Autor et al. (2014), Amiti et al. (2017), Pierce and Schott (2016), Feenstra et al. (2019), Borusyak and Jaravel (2019), and Sager and Jaravel (2019). A key contribution of this empirical research has been to provide compelling causal evidence on the effects of trade shocks using quasi-experimental variation. A continuing source of debate in implementing this empirical analysis is the appropriate measurement of trade shocks, including whether to focus on imports from one country, a group of countries or all countries; how to capture imports of final goods versus intermediate inputs; how to incorporate exports as well as imports; and how to measure third-market effects. Our research contributes to this debate by deriving theory-consistent measures of productivity and trade costs shocks that use only observed trade data, and that capture all of the above channels, including both partial and general equilibrium effects. As these sufficient statistic measures are derived from a class of theoretical models, they yield predictions for model-based objects such as welfare as well as for observed variables such as income.

Fifth, our analysis of countries' bilateral political attitudes is related to a large literature in economics, history and political science, including Scott (1955), Cohen (1960), Signorino and Ritter (1999), Alesina and Spolaore (2003), Martin et al. (2008), Kuziemko and Werker (2006), Guiso et al. (2009), Bao et al. (2019), and Häge (2011). We use our measures of exposure to productivity shocks to provide new evidence on the classic political economy question of the extent to which countries with shared economic interests also have similar political stances.

The remainder of the paper is structured as follows. Section 2 provides a characterization of the effects of productivity shocks in each country on income and welfare in all countries in an Armington model with a general homothetic utility function. Section 3 develops our measures of countries' income and welfare exposure to foreign productivity shocks for the special case of this model that falls within the class of models with a constant trade elasticity considered by Arkolakis et al. (2012), henceforth ACR. Section 4 reports a number of extensions and generalizations, including trade imbalances, small departures from a constant trade elasticity, multiple sectors following Costinot et al. (2012), henceforth CDK, and input-output linkages following Caliendo and Parro (2015), henceforth CP, and economic geography models with factor mobility. Section 5 reports our main empirical results for the impact of a productivity shock in one country on income and welfare in all countries. Section 6 provides empirical evidence on the extent to which countries that are economic friends of one another are also political friends. Section 7 concludes. A separate online appendix contains the derivations of the results in each section of the paper and the proofs of the propositions.

2 General Armington

We consider an Armington model with a general homothetic utility function, in which goods are differentiated by country of origin. We consider a world of many countries indexed by $n, i \in \{1, \ldots, N\}$. Each country has an exogenous supply of ℓ_n workers, who are each endowed with one unit of labor that is supplied inelastically.

2.1 Preferences

The representative consumer in country n has the following homothetic indirect utility function:

$$u_n = \frac{w_n}{\mathcal{P}\left(\boldsymbol{p}_n\right)}, \tag{1}$$

where $\boldsymbol{p}_n$ is the vector of prices in country n of the goods produced by each country i with elements p_{ni} (inclusive of trade costs); w_n is the wage; and $\mathcal{P}\left(\cdot\right)$ is a continuous and twice differentiable function that corresponds to the ideal price index for consumption. From Roy's Identity, country n's demand for the good produced by country i is:

$$c_{ni} = c_{ni}\left(\boldsymbol{p_n}\right) = -\frac{\partial\left(1/\mathcal{P}\left(\boldsymbol{p}_n\right)\right)}{\partial p_{ni}} w_n \mathcal{P}\left(\boldsymbol{p}_n\right). \tag{2}$$

2.2 Production

Each country's good is produced with labor according to a constant returns to scale production technology, with productivity z_i in country i. Markets are perfectly competitive. Goods can be traded between countries subject to iceberg trade costs, such that $\tau_{ni} \geq 1$ units of a good must be shipped from country i in order for one unit to arrive in country n (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Therefore, the cost in country n of consuming one unit of the good produced by country i is:

$$p_{ni} = \frac{\tau_{ni} w_i}{z_i}. \tag{3}$$

2.3 Expenditure Shares and Market Clearing

Country n's expenditure share on the good produced by country i can be written as:

$$s_{ni} = \frac{p_{ni} c_{ni}\left(\boldsymbol{p}_n\right)}{\sum_{\ell=1}^{N} p_{n\ell} c_{n\ell}\left(\boldsymbol{p}_n\right)}. \tag{4}$$

Totally differentiating this expenditure share equation, the proportional change in expenditure shares in country n depends on the proportional change in the prices of the goods from each country i and the own and cross-price elasticities for each good:

$$\mathrm{d}\ln s_{ni} = \sum_{h=1}^{N} \left[\theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nkh}\right] \mathrm{d}\ln p_{nh}, \tag{5}$$

where

$$\theta_{nih} \equiv \left(\frac{\partial\left(p_{ni} c_{ni}\left(\boldsymbol{p}_n\right)\right)}{\partial p_{nh}} \frac{p_{nh}}{p_{ni} c_{ni}\left(\boldsymbol{p}_n\right)}\right),$$

is the elasticity in country n of the expenditure share for good i with respect to the price of good h. Totally differentiating prices, the proportional change in the price in country n of the good produced by country i depends on the proportional changes in the underlying trade costs, wages and productivities as follows:

$$\mathrm{d}\ln p_{ni} = \mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i. \tag{6}$$

Market clearing requires that income in country i equals the expenditure on goods produced by that country:

$$w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n, \tag{7}$$

where for simplicity we begin by considering the case of balanced trade and show how the analysis generalizes to imbalanced trade in Section 4 below.

2.4 Comparative Statics

Using preferences (1) and market clearing (7), we now characterize the general equilibrium effect of shocks to productivity and trade costs. First, totally differentiating the market clearing condition (7) holding constant country endowments, the change in income in each country i depends on the share of value-added that it derives from each market n (t_{in}), the own and cross-price elasticities (θ_{nih}), and the proportional changes in the price of the good from each country h as determined by (6):

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + \left[\sum_{h=1}^{N} \left[\theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nkh}\right] \left[\mathrm{d}\ln \tau_{nh} + \mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right]\right]\right), \tag{8}$$

where the share of value-added that country i derives from each market n is defined as:

$$t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}. \tag{9}$$

Second, totally differentiating the indirect utility function (1), the change in welfare in country n equals the change in income in that country minus the expenditure share weighted average of the proportional change in the price of each country's good, as determined by (6):

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni} \left[\mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right]. \tag{10}$$

The market clearing condition for each country (8) shapes how exogenous changes in productivities ($\mathrm{d}\ln z_i$) and trade costs ($\mathrm{d}\ln \tau_{ni}$) map into endogenous changes in wages ($\mathrm{d}\ln w_i$). The utility function (10) determines how these endogenous changes in wages ($\mathrm{d}\ln w_i$) and the exogenous changes in productivities ($\mathrm{d}\ln z_i$) and trade costs ($\mathrm{d}\ln \tau_{ni}$) translate into endogenous changes in welfare in each country ($\mathrm{d}\ln u_n$). In general, both the own and cross-price elasticities of expenditure with respect to prices (θ_{nih}) are variable and depend on the entire price vector ($\boldsymbol{p}_n$), complicating the mapping from exogenous to endogenous variables.

3 Constant Elasticity of Import Demand

We now show that a sharp "friends" and "enemies" representation of countries' income and welfare exposure to foreign productivity or trade cost shocks can be obtained under the assumption of a constant trade elasticity. In Subsections 3.2 through 3.5, we derive this representation for small changes in productivity or trade costs under this assumption of a constant trade elasticity. In Subsection 3.6, we characterize the quality of the approximation for large changes as a function of the properties of the observed trade matrices, and show this approximation to be almost exact even for productivity shocks of the magnitude implied by the observed trade data.

Throughout this section, we derive our results in a single-sector, constant elasticity Armington model, which is a special case of the framework developed in the previous section. In Section E of the online appendix, we show that these results hold in the entire class of international trade models considered in Arkolakis et al. (2012), henceforth ACR, which satisfy the four primitive assumptions of (i) Dixit-Stiglitz preferences; (ii) one factor of production; (iii) linear cost functions; and (iv) perfect or monopolistic competition; as well as the three macro restrictions of (i) a constant elasticity import demand system, (ii) a constant share of profits in income, and (iii) balanced trade. In addition to the Armington model considered here, this class includes models of perfect competition and constant returns to scale with Ricardian technology differences, as in Eaton and Kortum (2002), and those of monopolistic competition and increasing returns to scale, in which goods are differentiated by firm, as in Krugman (1980) and Melitz (2003) with an untruncated Pareto productivity distribution.

While at the beginning of this section we allow for both productivity and trade cost shocks, we focus from subsection 3.3 onwards on productivity shocks alone. In Section 4 below, we consider a variety of extensions, including trade imbalances, trade cost shocks, multiple sectors following Costinot et al. (2012), input-output linkages following Caliendo and Parro (2015), and economic geography. We also derive sensitivity bounds for countries' income and welfare exposure to foreign productivity shocks for the more general case of a variable trade elasticity.

3.1 Trade Matrices

We begin by defining some notation. We use boldface, lowercase letters for vectors, and boldface, uppercase letters for matrices. We use the corresponding non-bold, lowercase letters for elements of vectors and matrices. We use $\mathbf{I}$ to denote the $N \times N$ identity matrix. We now introduce two key matrices of trade shares that we show below are central to determining the impact of productivity and trade cost shocks.

Expenditure Share and Income Share Matrices Let $\mathbf{S}$ be the $N \times N$ matrix with the ni-th element equal to importer n's expenditure on exporter i. Let $\mathbf{T}$ be the $N \times N$ matrix with the in-th element equal to the fraction of income that exporter i derives from selling to importer n. We refer to $\mathbf{S}$ as the *expenditure share* matrix and to $\mathbf{T}$ as the *income share* matrix. Intuitively, s_{ni} captures the importance of i as a *supplier* to country n, and t_{in} captures the importance of n as a *buyer* for country i. Note the order of subscripts: in matrix $\mathbf{S}$, rows are buyers and columns are suppliers, whereas in matrix $\mathbf{T}$, rows are suppliers and columns are buyers. Both matrices have rows that sum to one.

These $\mathbf{S}$ and $\mathbf{T}$ matrices are equilibrium objects that can be obtained directly from observed trade data. We derive comparative statics results using these observed matrices. Using $\mathbf{S}^k$ to represent the matrix $\mathbf{S}$ raised to the k-th power, we impose the following technical assumption on the matrix $\mathbf{S}$, which is satisfied in the observed trade flow data.

Assumption 1. *(i) For any i, n, there exists k such that $\left[\mathbf{S}^k\right]_{in} > 0$. (ii) For all i, $s_{ii} > 0$.*

The first part of this assumption states that all countries trade with each other *directly* or *indirectly*. That is, in the language of graph theory, the global trade network is *strongly connected*. This assumption is important because shocks propagate in general equilibrium through changes in relative prices, which are only well-defined if countries are connected (potentially indirectly) to each other through trade. When the global trade network has disconnected components—for instance, if a subset of countries only trade among themselves but not with other nations, or if some countries are in autarky—our results can be applied to study the general equilibrium propagation of shocks within each of the connected components separately. In practice, we find that the global trade network is strongly connected throughout our sample period. The second part of this assumption ensures that every country consumes a positive amount of domestic goods, which again is satisfied in all years.

Using Assumption 1, we now establish a relationship between the $\mathbf{S}$ and $\mathbf{T}$ matrices, which shapes the general equilibrium impact of productivity shocks on income and welfare.

Lemma 1. *Assuming that trade is balanced,*

1. *$\mathbf{S}$ has a unique left-eigenvector $\mathbf{q}'$ with all positive entries summing to one; the corresponding eigenvalue is one.*
2. *The i-th element of this left-eigenvector q_i is the equilibrium income of country i relative to world nominal GDP, $q_i = w_i \ell_i / \left(\sum_{n=1}^{N} w_n \ell_n\right)$.*
3. *$\mathbf{q}'$ is also a left-eigenvector of $\mathbf{T}$ with eigenvalue one, and $q_i t_{in} = q_n s_{ni}$.*
4. *Under free-trade (i.e. $\tau_{ni} = 1$ for all n, i), $\mathbf{q}'$ is equal to every row of $\mathbf{S}$ and of $\mathbf{T}$.*

Proof. See Section B.1 of the online appendix. □

Going forward, we refer to the vector $\mathbf{q}'$ as simply the income vector, reflecting our normalization that world nominal GDP is equal to one. Lemma 1 shows that, under balanced trade, one could recover $\mathbf{q}$ and $\mathbf{T}$ from the expenditure share matrix $\mathbf{S}$. A key implication of this result is that $\mathbf{S}$ is a sufficient statistic for the general equilibrium effect of small productivity shocks on income and welfare under balanced trade.[2]

In the remainder of this section, we use these properties of the trade matrices to characterize the first-order general equilibrium effects of global productivity shocks on income and welfare in each country in the constant elasticity version of the Armington model developed in Section 2 above.

[2] As the expenditure and income shares sum to one, both the matrices $\mathbf{S}$ and $\mathbf{T}$ represent row-stochastic Markov chains, and $\mathbf{q}'$ is their stationary distribution. Assumption 1 ensures that the matrix $\mathbf{S}$ is *primitive.* Since the elements of the matrix $\mathbf{T}$ satisfy $q_i t_{in} = q_n s_{ni}$, the Markov chain $\mathbf{S}$ is *reversible* if and only if $\mathbf{S} = \mathbf{T}$, which holds if and only if trade is balanced bilaterally between each country-partner-pair, a condition which is not satisfied in the data. Finally, the matrix $\mathbf{TS}$, which we show below determines the cross-price elasticity under a constant trade elasticity, is the *multiplicative reversiblization* of $\mathbf{S}$ (Fill 1991), with $q_i \left[\mathbf{TS}\right]_{in} = q_n \left[\mathbf{TS}\right]_{ni}$. Note that the income vector $\mathbf{q}'$ is a left-eigenvector of this matrix $\mathbf{TS}$ with eigenvalue one.

3.2 First-Order Comparative Statics

In the constant elasticity Armington specification, the preferences of the representative consumer in country n in equation (1) are characterized by the following functional form:

$$u_n = \frac{w_n}{\left[\sum_{i=1}^{N} p_{ni}^{-\theta}\right]^{-\frac{1}{\theta}}}, \qquad \theta = \sigma - 1, \qquad \sigma > 1, \tag{11}$$

where $\sigma > 1$ is the constant elasticity of substitution between country varieties and $\theta = \sigma - 1$ is the trade elasticity. Using Roy's Identity, country n's share of expenditure on the good produced by country i is:

$$s_{ni} = \frac{p_{ni}^{-\theta}}{\sum_{m=1}^{N} p_{nm}^{-\theta}}. \tag{12}$$

Using these functional forms in the market clearing condition (7) and totally differentiating holding constant country endowments, the system of equations for the change in income (8) now simplifies to:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} \begin{array}{c} s_{nh}\left[\mathrm{d}\ln \tau_{nh} + \mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] \\ -\left[\mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] \end{array} \right) \right). \tag{13}$$

The system of equations for the change in welfare again takes the same form as in equation (10):

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni}\left[\mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right]. \tag{14}$$

Given exogenous changes in productivities ($\mathrm{d}\ln z_i$) and trade costs ($\mathrm{d}\ln \tau_{ni}$), the market clearing condition for each country (8) provides a system of N equations that can be used to determine the N endogenous changes in wages in each country ($\mathrm{d}\ln w_i$). Combining these endogenous changes in wages ($\mathrm{d}\ln w_i$) with the exogenous changes in productivities ($\mathrm{d}\ln z_i$) and trade costs ($\mathrm{d}\ln \tau_{ni}$), the utility function (10) determines the N endogenous changes in welfare in each country ($\mathrm{d}\ln u_n$).

3.3 Sufficient Statistics for Income and Welfare Exposure to Productivity Shocks

We now use these comparative statics results in equations (13) and (14) to obtain our "friend" and "enemy" sufficient statistics for countries' income and welfare exposure to foreign shocks. To streamline the exposition and in light of our empirical application, we now focus on productivity shocks ($\mathrm{d}\ln z_i \neq 0$), assuming that trade cost shocks are zero ($\mathrm{d}\ln \tau_{ni} = 0 \ \forall n, i$). In Subsection 4.1 in the next section, we show that our approach naturally also accommodates trade cost shocks ($\mathrm{d}\ln \tau_{ni} \neq 0$).

3.3.1 Sufficient Statistics for Income Levels

We begin by showing that the first-order general equilibrium effects of small productivity shocks in each country on income in all countries in equation (13) have a matrix representation, which has two key advantages for our purposes. First, we can use this representation to recover our "friend" and "enemy" measures of countries' exposure to a foreign productivity shock as a simple matrix inversion problem, which can be solved almost instantaneously to machine precision. Second, this representation isolates key mechanisms in the model that enable us to relate its quantitative predictions to underlying economic forces. Using $\mathrm{d}\ln \mathbf{z}$ and $\mathrm{d}\ln \mathbf{w}$ to denote column vectors of country-level productivity shocks and wage responses, we have the following matrix representation of equation (13).

Proposition 1. *Under ACR assumptions (i)-(iv) and macro restrictions (i)-(iii), the first-order general equilibrium impact of productivity shocks on income in all countries around the world solves the fixed point equation:*

$$\underbrace{\mathrm{d}\ln \mathbf{w}}_{\text{income effect}} = \underbrace{\mathbf{T}\,\mathrm{d}\ln \mathbf{w}}_{\text{market-size effect}} + \underbrace{\theta \cdot \mathbf{M} \times (\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z})}_{\text{cross-substitution effect}}, \tag{15}$$

where $\mathbf{M} \equiv \mathbf{TS} - \mathbf{I}$ *is an* $N \times N$ *matrix with* in*-th entry* $m_{in} \equiv \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i}$.

Proof. The proposition follows immediately from equation (13) and our assumption that that $\mathrm{d}\ln \tau_{ni} = 0\ \forall n, i$, as shown in Section D of the online appendix. □

From equation (15), we can compute the effect of small productivity shocks on income in each country around the world using the income share matrix ($\mathbf{T}$) and the cross-substitution matrix ($\mathbf{M}$), both of which are transformations of the expenditure share matrix ($\mathbf{S}$). The matrix $\mathbf{T}$ in the first term on the right-hand side captures a *market-size effect*: To the extent that the productivity shock vector $\mathrm{d}\ln \mathbf{z}$ increases incomes in countries n, this raises income in country i through increased demand for its goods. In particular, the elements of $\mathbf{T}$ are the share of income that country i earns through selling to each market n (t_{in}), and capture how dependent country i is on markets in each country n.

The matrix $\mathbf{M}$ in the second term on the right-hand side captures a *cross-substitution effect*. To understand this effect, consider the in-th element of this matrix: $m_{in} \equiv \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i}$. For $i \neq n$, the sum $\sum_{h=1}^{N} t_{ih} s_{hn}$ captures the overall competitive exposure of country i to country n, through each of their common markets h, weighted by the importance of market h for country i's income (t_{ih}). As the competitiveness of country n increases, as measured by a decline in its wage relative to its productivity ($\mathrm{d}\ln w_n - \mathrm{d}\ln z_n$), consumers in all markets h substitute towards country n and away from other countries $i \neq n$, thereby reducing income in country i and raising it in country n. With a constant elasticity import demand system, the magnitude of this cross-substitution effect in market h depends on the trade elasticity (θ) and the share of expenditure in market h on the goods produced by country n (s_{hn}): consumers in market h increase expenditure on country n by $(s_{hn} - 1)$ and lower expenditure on country i by s_{hn}. Summing across all markets h, we obtain the overall impact of the shock to country n's production cost on country i's income, as captured in the in-th element of the matrix $\mathbf{M}$.

We now use this matrix representation in Proposition 1 to recover our "friend" and "enemy" measures of countries' bilateral income exposure to productivity growth. As the trade share matrices $\mathbf{T}$ and $\mathbf{M}$ in equation (15) are homogenous of degree zero in incomes, they do not pin down the level of changes in nominal incomes. As in any general equilibrium model, we need a choice of numeraire. We choose world GDP as our numeraire, which with unchanged country endowments (ℓ_i) implies the following normalization: $\sum_{i=1}^{N} q_i \,\mathrm{d}\ln w_i = 0$. Starting with equation (15), dividing both sides by $(\theta + 1)$, re-arranging terms, and using this normalization, we obtain:

$$(\mathbf{I} - \mathbf{V})\ \mathrm{d}\ln w = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln \mathbf{z}, \qquad \mathbf{V} \equiv \frac{\mathbf{T} + \theta \mathbf{TS}}{\theta + 1} - \mathbf{Q}, \tag{16}$$

where $\mathbf{Q}$ is an $N \times N$ matrix with the income row vector $\mathbf{q}'$ stacked N times and recall that we assume $\theta > 0$. Under free-trade (i.e. $\tau_{ni} = 0$ for all n, i), $\mathbf{Q} = \mathbf{S} = \mathbf{T}$.

The presence of the term $\mathbf{Q}\,\mathrm{d}\ln \mathbf{w} = \mathbf{0}$ on the left-hand side in equation (16) reflects our choice of numeraire. In the absence of this term, the matrix $\left(\mathbf{I} - \frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}\right)$ is not invertible: the income shares and expenditure shares sum to one ($\sum_{n=1}^{N} t_{in} = 1$ and $\sum_{n=1}^{N} s_{ni} = 1$), thus the rows of $\frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}$ also sum to one, and the columns of $\left(\mathbf{I} - \frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}\right)$ are not linearly independent. This non-invertibility reflects the fact that income can only be recovered from expenditure shares up to a normalization or choice of units. While we choose world GDP as our numeraire because it is convenient for the matrix inversion,[3] all of our predictions for relative country incomes are invariant to whatever normalization is chosen. Using equation (16), we are now in a position to formally state the following definition.

Definition 1. Our *friends-and-enemies matrix for income* is defined as:

$$\mathbf{W} \equiv -\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\mathbf{M}. \tag{17}$$

From Definition 1 and equation (16), our friends-and-enemies matrix $\mathbf{W}$ completely summarizes the general equilibrium effect of small productivity shocks on income in each country around the world.

Corollary 1. *Income exposure to global productivity shocks is:*

$$\mathrm{d}\ln\mathbf{w} = \mathbf{W}\,\mathrm{d}\ln\mathbf{z} \tag{18}$$

Proof. The corollary follows immediately from Proposition 1, Definition 1, and our choice of world GDP as numeraire ($\mathbf{Q}\,\mathrm{d}\ln\mathbf{w} = \mathbf{0}$), as shown in Section D of the online appendix. □

The elements of this matrix $\mathbf{W}$ capture countries' bilateral income exposure to productivity shocks. In particular, the in-th element of this matrix is the elasticity of income in country i (row) with respect to a small productivity shock in country n (column). We refer to country n as being a "friend" of country i for income when this elasticity is positive and an "enemy" of country i for income when this elasticity is negative. In general, $\mathbf{W}$ is not necessarily symmetric: i could view n as a friend, while n views i as an enemy. Finally, we now establish that the friends-and-enemies matrix $\mathbf{W}$ in Definition 1 exists, because the matrix $(I-V)$ is invertible under Assumption 1.

Lemma 2. *Let* $\mathbf{V} \equiv \frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1} - \mathbf{Q}$. *Under Assumption 1 and* $\theta > 0$, *the matrix* $(\mathbf{I}-\mathbf{V})$ *is invertible,* $(\mathbf{I}-\mathbf{V})^{-1} = \sum_{k=0}^{\infty}\mathbf{V}^k$, *and the power series converge at rate* $|\mu| < 1$, *where* $|\mu|$ *is the absolute value of the largest eigenvalue of* $\mathbf{V}$ *(i.e.,* $||\mathbf{V}^k|| \leq c\cdot|\mu|^k$ *for some constant* c*).*

Proof. See Section B.2 of the online appendix. □

Therefore, given the observed trade matrices ($\mathbf{S}$, $\mathbf{T}$ and $\mathbf{M}$), we obtain a complete characterization of the general equilibrium effect of small productivity shocks under our assumption of a constant trade elasticity.

3.4 Sufficient Statistics for Welfare

We next show that the general equilibrium effects of small productivity shocks in all countries on welfare in each country in equation (14) have an analogous matrix representation, which again allows us to connect quantitative predictions directly to underlying economic mechanisms in the model. Using $\mathrm{d}\ln\mathbf{u}$ to denote the column vector of country-level welfare changes, we have the following matrix representation of equation (14).

[3] Note that the matrix $\frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}$ represents a row-stochastic Markov chain; its left eigenvector $\mathbf{q}'$ is also the stationary distribution of the Markov chain, and $\lim_{k\to\infty}\left(\frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}\right)^k = \mathbf{Q}$.

Proposition 2. *Under ACR assumptions (i)-(iv) and macro restrictions (i)-(iii), the first-order general equilibrium impact of productivity shocks on welfare in all countries around the world solves the following fixed point equation:*

$$\underbrace{\mathrm{d}\ln\mathbf{u}}_{\textit{welfare effect}} = \underbrace{\mathrm{d}\ln\mathbf{w}}_{\textit{income effect}} - \underbrace{\mathbf{S}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right)}_{\textit{cost-of-living effect}}. \tag{19}$$

Proof. The proposition follows immediately from equation (14) and our assumption that $\mathrm{d}\ln\tau_{ni} = 0\;\forall n, i$, as shown in Section D of the online appendix. □

From Propositions 1 and 2, we can compute the effect of productivity shocks on the welfare of all countries around the world using the income share matrix ($\mathbf{T}$), the cross-substitution matrix ($\mathbf{M}$), and the expenditure share matrix ($\mathbf{S}$), where both the income share and cross-substitution matrices are transformations of the expenditure share matrix. The presence of this expenditure share matrix ($\mathbf{S}$) in the second term on the right-hand side of equation (19) captures a cost of living effect, which reflects the impact of the productivity shock in country i on the price index in country n. The elements of this matrix s_{ni} capture the relative importance of each country i in the consumer expenditure bundle of country n. A productivity shock in country i will have a large positive effect on welfare in country n if it has a large positive effect on wages in country n (through the income effect) and a large negative effect on wages and production costs in the countries from which country n sources most of its goods (through the cost of living effect).

As the elements of the expenditure share matrix ($\mathbf{S}$) are homogeneous of degree zero in per capita income and sum to one for each importer, adding any constant vector $\mathbf{c}$ to changes in log per capita incomes ($\mathrm{d}\ln\mathbf{w} = \mathrm{d}\ln\mathbf{w} + \mathbf{c}$) leaves the welfare effect in equation (19) unchanged (since $\mathbf{c} - \mathbf{S}\mathbf{c} = \mathbf{0}$). Therefore, the welfare effect in Proposition 2 is invariant to our choice of numeraire. Using Corollary 1 and Proposition 2, we are now in a position to formally state the following definition.

Definition 2. Our *friends-and-enemies matrix for* welfare is defined as:

$$\mathbf{U} \equiv (\mathbf{I} - \mathbf{S})\,\mathbf{W} + \mathbf{S}. \tag{20}$$

From Definition 2, our friends-and-enemies matrix $\mathbf{U}$ completely summarizes the general equilibrium effect of small productivity shocks on welfare in each country around the world.

Corollary 2. *Welfare exposure to global productivity shocks is:*

$$\mathrm{d}\ln\mathbf{u} = \mathbf{U}\,\mathrm{d}\ln\mathbf{z}. \tag{21}$$

Proof. The corollary follows immediately from Corollary 1, Definition 2 and Proposition 2. □

The elements of this matrix $\mathbf{U}$ capture countries' bilateral welfare exposure to productivity shocks. In particular, the ni-th element of this matrix is the elasticity of welfare in country n (row) with respect to a small productivity shock in country i (column). We refer to country i as being a "friend" of country n for welfare when this elasticity is positive and an "enemy" of country n for welfare when this elasticity is negative. As for income exposure, welfare exposure $\mathbf{U}$ is not necessarily symmetric: i could view n as a friend, while n views i as an enemy.

3.5 Economic Mechanisms

We now use our friends-and-enemies matrix representation to isolate the key economic mechanisms through which a productivity shock in one country affects income and welfare in all countries in this class of international trade models with a constant trade elasticity.

1. Partial and General Equilibrium Effects Our measure of the overall impact of foreign productivity shocks on domestic income in equation (17) includes both the direct (partial equilibrium) effect of these productivity shocks ($\mathrm{d}\ln\mathbf{z}$) on competitiveness in each market, as well as their indirect (general equilibrium) effects on competitiveness and the size of each market through endogenous changes in incomes ($\mathrm{d}\ln\mathbf{w}$). To separate these two effects, we use the property that the spectral radius of $\mathbf{V}$ is less than one, which allows us to re-write our income exposure measure as the following power series:

$$\mathbf{W} = -\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\mathbf{M} = -\frac{\theta}{\theta+1}\sum_{k=0}^{\infty}\mathbf{V}^{k}\mathbf{M} = -\underbrace{\frac{\theta}{\theta+1}\mathbf{M}}_{\text{partial equilibrium}} \underbrace{-\frac{\theta}{\theta+1}\left(\mathbf{V}+\mathbf{V}^{2}+\dots\right)\mathbf{M}}_{\text{general equilibrium}}, \tag{22}$$

where the first term on the right-hand side ($-\frac{\theta}{\theta+1}M$) captures the partial equilibrium effect (the direct effect of higher productivity in country ℓ on income in country i, evaluated at the initial values of incomes in each country); the second term on the right-hand side ($-\frac{\theta}{\theta+1}\mathbf{VM}$) and the following higher-order terms in $\mathbf{V}$ capture the general equilibrium effect (through endogenous changes in incomes in each country).[4]

2. Market-Size and Cross-Substitution Effects From Proposition 1, overall income exposure to productivity shocks is jointly determined by the market-size and cross-substitution effects. To separate out the contributions of each of these mechanisms to general equilibrium changes in incomes, we undertake a counterfactual exercise in which we impose that the market-size effect is the same for all countries and allow only the cross-substitution effect to differ across countries. Specifically, we replace the term $\mathbf{T}\,\mathrm{d}\ln\mathbf{w}$ in equation (15) with $\mathbf{Q}\,\mathrm{d}\ln\mathbf{w}$, so that the general equilibrium income response to productivity shocks $\mathbf{d}\ln\mathbf{w}^{\text{Sub}}$ solves the fixed point equation:

$$\mathrm{d}\ln\mathbf{w}^{\text{Sub}} = \mathbf{Q}\,\mathrm{d}\ln\mathbf{w}^{\text{Sub}} + \theta\cdot\mathbf{M}\left(\mathrm{d}\ln\mathbf{w}^{\text{Sub}} - \mathrm{d}\ln\mathbf{z}\right), \tag{23}$$

where we use the superscript Sub to indicate cross-substitution effect.

In our actual income exposure measure in equation (15), the rows of the matrix $\mathbf{T}$ vary across countries i with the shares of markets in their income ($\mathbf{t}'$). In contrast, in this counterfactual income exposure measure in equation (23), the rows of the matrix $\mathbf{Q}$ are the same across countries i and equal to the shares of markets in world income ($\mathbf{q}'$). Using our choice of world GDP as numeraire ($\mathbf{Q}\,\mathrm{d}\ln\mathbf{w} = \mathbf{0}$), we can recover counterfactual income exposure from the cross-substitution effect alone from the following matrix inversion:

$$\mathbf{W}^{\text{Sub}} \equiv -\theta\left(\mathbf{I}-\theta\left(\mathbf{M}+\mathbf{Q}\right)\right)^{-1}\mathbf{M}. \tag{24}$$

$$\mathrm{d}\ln\mathbf{w}^{\text{Sub}} = \mathbf{W}^{\text{Sub}}\,\mathrm{d}\ln\mathbf{z}. \tag{25}$$

[4] Here we define the direct or partial equilibrium effect for a given country as holding all other countries incomes constant at their values in the initial equilibrium before the productivity shock, but other definitions are possible, as discussed in a different context in Huo et al. (2019).

While our friends-and-enemies matrix for income $\mathbf{W}$ from the previous subsection captures overall income exposure to productivity shocks; the matrix $\mathbf{W}^{\text{Sub}} \equiv -\theta \mathbf{M} \left(\mathbf{I} - \theta \left(\mathbf{M} + \mathbf{Q}\right)\right)^{-1}$ captures income exposure through the cross-substitution effect alone; and the difference $\mathbf{W} - \mathbf{W}^{\text{Sub}}$ captures income exposure through the market-size effect.

3. Contribution of Third Markets to Bilateral Income Exposure Our measures of exposure to productivity shocks capture all mechanisms through which productivity shocks affect income and welfare in the model, including both imports and exports and both own and third-market effects. We now use our approach to evaluate how much of one country's exposure to productivity shocks operates through third markets. Let $\mathcal{G}$ denote the subset of countries for which we are interested in third-market effects (e.g. for U.S. income exposure to a Chinese productivity shocks, $\mathcal{G}$ might be the European Union). To evaluate the contribution of these third markets to income and welfare exposure, we construct counterfactual expenditure share matrices excluding them.

In particular, we define $\mathbf{S}_{-\mathcal{G}}$ as the transformed expenditure share matrix, removing the k-th rows and columns from $\mathbf{S}$ for all $k \in \mathcal{G}$, and rescaling the remaining rows to sum to one. Using this counterfactual expenditure share matrix $\mathbf{S}_{-\mathcal{G}}$, we construct the corresponding income share matrix $\mathbf{T}_{-\mathcal{G}}$ and cross-substitution matrix $\mathbf{M}_{-\mathcal{G}}$. Using these counterfactual trade share matrices, we recompute both our overall measure of income exposure ($\mathbf{W}_{-\mathcal{G}}$) using equation (17) and the cross-substitution effect ($\mathbf{W}^{\text{Sub}}_{-\mathcal{G}}$) using equation (24). Comparing these measures to those including all countries ($\mathbf{W}$, $\mathbf{W}^{\text{Sub}}$), we can quantify the importance of this group of third markets for both overall income exposure and the cross-substitution effect.

Finally, welfare exposure ($\mathbf{U}$) in Definition 2 is a linear combination of income exposure ($\mathbf{W}$) and the expenditure share matrix ($\mathbf{S}$) that controls the cost of living effect. Therefore, substituting each of the above decompositions of income exposure ($\mathbf{W}$) into welfare exposure ($\mathbf{U}$), we can quantify the contribution of each of these mechanisms to the impact of productivity shocks on welfare.

3.6 Comparison with Exact Hat-Algebra

Our friends-and-enemies exposure measures have the advantage that they are quick and easy to compute using only matrices of observed trade data. They also allow researchers working with quantitative trade models to transparently assess the role of different economic mechanisms. A potential limitation is that our exposure measures correspond to first-order effects in a linearization that is only exact for small changes, which raises the question of how good an approximation they provide for large changes. We now characterize the quality of this approximation by relating the magnitude of the second and higher-order terms in the Taylor-series expansion to properties of the observed trade matrices. In our later empirical analysis, we use these results to show that our linearization is almost exact for productivity shocks, even for large changes of the magnitude implied by the observed trade data.

We begin by comparing our linearization to the full non-linear solution of the model for large changes using the exact-hat algebra approach of Dekle et al. (2007). In particular, using this exact-hat algebra approach, we can re-write the market clearing condition (7) in a counterfactual equilibrium following a productivity shock (denoted by a prime) in terms of the observed values of variables in an initial equilibrium (no prime) and the relative changes of variables between the counterfactual and initial equilibria (denoted by a hat such, that $\hat{x} = x'/x$):

$$\ln \hat{w}_i = \left(\frac{\theta}{\theta+1}\right) \ln \hat{z}_i + \frac{1}{\theta+1} \ln \left[\sum_{n=1}^{N} t_{in} \frac{\hat{w}_n}{\sum_{\ell=1}^{N} s_{n\ell} \hat{w}_\ell^{-\theta} \hat{z}_\ell^{\theta}}\right], \tag{26}$$

which provides a system of N equations that can be solved for the N unknown relative changes in wages ($\hat{w}_n$) given the assumed productivity shock ($\hat{z}_\ell$) and the observed trade shares (t_{in}, s_{ni}) in the initial equilibrium.

Using equation (15), we can re-write our friends-and-enemies income exposure measure in the following similar but log linear form:

$$\mathrm{d}\ln w_i = \left(\frac{\theta}{\theta+1}\right)\mathrm{d}\ln z_i + \frac{1}{\theta+1}\sum_{n=1}^{N} t_{in}\left[\mathrm{d}\ln w_n + \theta\sum_{\ell=1}^{N} s_{n\ell}\left[\mathrm{d}\ln w_\ell - \mathrm{d}\ln z_\ell\right]\right]. \tag{27}$$

Comparing equations (26) and (27), we find that the difference between the predictions of the exact-hat algebra and our friend-enemy linearization corresponds to the difference between the log of a weighted mean and a weighted mean of logs. These two expressions take the same value as trade costs become large ($t_{nn} \to 1$, $s_{nn} \to 1$ for all n) and under free trade. More generally, these two expressions take different values, with the difference between them equal to the second and higher-order terms in a Taylor-series expansion. We now characterize the properties of the second-order term in this expansion, before bounding the magnitude of all higher-order terms. To simplify notation, we define $\tilde{z}_i$ as $\ln \hat{z}_i$. We use $f_i(\tilde{\mathbf{z}})$ to denote the implicit function that defines the log changes in wages $\tilde{w}_i$ in equation (26) as a function of the log productivity shocks $\{\tilde{\mathbf{z}}\}$, and we use $\epsilon_i(\tilde{\mathbf{z}})$ to denote the second-order term in the Taylor-series expansion of $f_i(\tilde{\mathbf{z}})$. Using this notation, we can rewrite equation (26) as:

$$\tilde{w}_i \quad = \quad \underbrace{-\theta\left(\tilde{w}_i - \tilde{z}_i\right) + \sum_n t_{in}\tilde{w}_n + \theta\sum_n m_{in}\left[\tilde{w}_n - \tilde{z}_n\right] +}_{\text{first-order}} \underbrace{\epsilon_i\left(\tilde{\mathbf{z}}\right)}_{\text{second-order}} \underbrace{+O\left(\|\tilde{\mathbf{z}}\|^3\right)}_{\text{higher-order}}.$$

The properties of the second-order term depend on the Hessian $\mathbf{H}_{f_i}$ of the function f_i evaluated at $\tilde{z}_\ell = 0\ \forall\ \ell$:

$$\mathbf{H}_{f_i} \equiv \begin{bmatrix} \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_1^2} & \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_1 \partial \tilde{z}_2} & \cdots & \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_1 \partial \tilde{z}_N} \\ \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_2 \partial \tilde{z}_1} & \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_2^2} & \cdots & \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_2 \partial \tilde{z}_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_N \partial \tilde{z}_1} & \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_N \partial \tilde{z}_2} & \cdots & \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_N^2} \end{bmatrix}, \tag{28}$$

where we can write this second-order term as $\epsilon_i(\tilde{\mathbf{z}}) = \tilde{\mathbf{z}}'\mathbf{H}_{f_i}\tilde{\mathbf{z}}$.

We now proceed as follows. First, we derive an expression for this Hessian in terms of matrices of observed trade data (Proposition 3). Second, we show that a cross-country average of the second-order terms is exactly zero (Proposition 4). Third, we show that the absolute magnitude of this second-order term for each country can be bounded by the largest eigenvalue (in absolute) value of this Hessian (Proposition 5). As this largest eigenvalue can be measured using observed trade data, we can use this result to bound the quality of the approximation for each country given the observed trade matrices. Fourth, we aggregate these results for the second-order terms across countries, and provide an upper bound on their sums of squares (Proposition 6). Again this bound can be computed using observed trade data and provides a summary measure of the overall performance of our linearization. Finally, Proposition 7 provides a bound on *all* higher order terms, including the second-order term and beyond.

In Proposition 3, we show that the Hessian ($\mathbf{H}_{f_i}$) depends solely on the trade elasticity (θ) and the three observed matrices that capture the market-size effects ($\mathbf{T}$), cross-substitution effects ($\mathbf{M}$), and expenditure shares ($\mathbf{S}$). In particular, the second-order term depends on expectations and variances taken across the elements of these matrices, as summarized in the following proposition.

Proposition 3. *The Hessian matrix can be explicitly written as*

$$\mathbf{H}_{f_i} \quad = \quad -\frac{1}{2}\left(\mathbf{A}'\left(diag\left([\mathbf{M}+\mathbf{I}]_i\right) - \mathbf{S}'diag\left(\mathbf{T}_i\right)\mathbf{S}\right)\mathbf{A} - \mathbf{B}'\left(diag\left(\mathbf{T}_i\right) - \mathbf{T}_i'\mathbf{T}_i\right)\mathbf{B}\right).$$

where $\mathbf{A} \equiv \frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right)$ *and* $\mathbf{B} \equiv \frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\mathbf{M}+\mathbf{SA}$*, and* $\mathbf{T}_i$*,* $\mathbf{M}_i$ *are the* i*-th rows of* $\mathbf{T}$ *and* $\mathbf{M}$*, respectively.*

The second-order term $\epsilon_i\left(\tilde{\mathbf{z}}\right) \equiv \tilde{\mathbf{z}}'\mathbf{H}_{f_i}\tilde{\mathbf{z}}$ *can be re-written more intuitively as*

$$\epsilon_i\left(\tilde{\mathbf{z}}\right) = -\frac{\theta^2\mathbb{E}_{T_i}\mathbb{V}_{S_n}\left[\ln\hat{w}_k - \tilde{z}_k\right]}{2} + \frac{\mathbb{V}_{T_i}\left(\ln\hat{w}_i + \theta\mathbb{E}_{S_n}\left[\ln\hat{w}_k - \tilde{z}_k\right]\right)}{2},$$

where $\mathbb{E}_{T_i}$*,* $\mathbb{E}_{M_i}$*,* $\mathbb{E}_{S_n}$*,* $\mathbb{V}_{T_i}$*, and* $\mathbb{V}_{S_n}$ *are expectations and variances taken using* $\{\mathbf{T}_{in}\}_{n=1}^N$*,* $\{\mathbf{M}_{in}\}_{n=1}^N$*, and* $\{\mathbf{S}_{nk}\}_{k=1}^N$ *as measures (e.g.* $\mathbb{E}_{T_i}\left[x_n\right] \equiv \sum_{n=1}^N \mathbf{T}_{in}x_n$*,* $\mathbb{V}_{T_i}\left[x_n\right] \equiv \sum_{n=1}^N \mathbf{T}_{in}x_n^2 - \left(\sum_{n=1}^N \mathbf{T}_{in}x_n\right)^2$*).*

Proof. See Section B.3 of the online appendix. □

As a first step towards characterizing the magnitude of the second-order terms in this expression, we next show in Proposition 4 that the average across countries (weighted by country size in the initial equilibrium before the productivity shock) of these second-order terms is exactly zero: $\mathbf{q}'\epsilon\left(\tilde{\mathbf{z}}\right) = 0$. Therefore, these second-order terms raise or reduce the predicted change in the wage of individual countries in response to the productivity shock, but when weighted appropriately they average out across countries.

Proposition 4. *Weighted by each country's income, the second-order terms average to zero for any productivity shock vector:* $\mathbf{q}'\epsilon\left(\tilde{\mathbf{z}}\right) = 0$ *for all* $\tilde{\mathbf{z}}$*.*

Proof. See Section B.4 of the online appendix. □

We now bound the absolute value of the second-order term for the income response of each country, following any vector of productivity shocks. First, note that because the model features constant returns to scale, a uniform shock to the productivity of all countries across the globe does not affect relative income. It is therefore without loss of generality to focus on productivity shocks that average to zero. We now show in Proposition 5 that the absolute value of the second-order term for the log-change in income of each country i is bounded, relative to the variance of productivity shocks, by the largest eigenvalue $\mu^{\max,i}$ (by absolute value) of the Hessian matrix $\mathbf{H}_{f_i}$ ($\left|\epsilon_i\left(\tilde{\mathbf{z}}\right)\right| \leq \left|\mu^{\max,i}\right| \cdot \tilde{\mathbf{z}}^T\tilde{\mathbf{z}}$). The corresponding eigenvector $\tilde{\mathbf{z}}^{\max,i}$ is the productivity shock vector that achieves the largest second-order term for country i. As these eigenvalues of the Hessian matrix for each country can be evaluated using the observed trade matrices, we thus obtain a bound on the size of second-order term for each country that can be computed in practice using the observed trade data. In our empirical application below, we show that for each country, even the largest eigenvalue is close to zero, which in turn implies that the second-order term for each country is close to zero.

Proposition 5. $\left|\epsilon_i\left(\tilde{\mathbf{z}}\right)\right| \leq \left|\mu^{\max,i}\right| \cdot \tilde{\mathbf{z}}'\tilde{\mathbf{z}}$ *for all* $\tilde{\mathbf{z}}$*, where* $\mu^{\max,i}$ *is the largest eigenvalue of* $\mathbf{H}_{f_i}$ *by absolute value. Let* $\tilde{\mathbf{z}}^{\max,i}$ *denote the corresponding eigenvector (such that* $\mathbf{H}_{f_i}\tilde{\mathbf{z}}^{\max,i} = \mu^{\max,i}\tilde{\mathbf{z}}^{\max,i}$*). The upper bound for* $\left|\epsilon_i\left(\tilde{\mathbf{z}}\right)\right|$ *is achieved when productivity shocks are represented by* $\tilde{\mathbf{z}}^{\max,i}$*:* $\left|\epsilon_i\left(\tilde{\mathbf{z}}^{\max,i}\right)\right| = \left|\mu^{\max,i}\right| \cdot \left(\tilde{\mathbf{z}}^{\max,i}\right)^T\tilde{\mathbf{z}}^{\max,i}$*.*

Proof. See Section B.5 of the online appendix. □

We next aggregate the second-order terms across countries and provide an upper-bound on their sum-of-squares in Proposition 6, which enables us to assess the overall performance of our linear approximation. As we show in our empirical application later, the standard unit vector $\mathbf{e}^\ell$ comes close to achieving the upper-bound for the ℓ-th equation, i.e. $\mathbf{e}^\ell \approx \tilde{\mathbf{z}}^{\max,\ell}$ for all ℓ. Intuitively, because $\mathbf{e}^i$ is orthogonal to $\mathbf{e}^j$ for all $i \neq j$, this implies that the productivity shock vectors $\tilde{\mathbf{z}}^{\max,i}$ and $\tilde{\mathbf{z}}^{\max,j}$ that maximize second-order effects for different countries $i \neq j$ are almost orthogonal. Hence, given any productivity shock vector $\tilde{\mathbf{z}}$, at most one country $\ln \hat{w}_i = f_i(\tilde{\mathbf{z}})$ can have a second-order term close to the upper-bound $\mu^{\max,i}$, which is small, and the second-order terms for all other countries are close to zero. To formalize this intuition, Proposition 6 constructs a symmetric order-4-tensor $\mathcal{A}$ such that $\frac{\sqrt{\frac{1}{N}\sum_{i=1}^N \epsilon_i^2(\tilde{\mathbf{z}})}}{\tilde{\mathbf{z}}^T\tilde{\mathbf{z}}}$ is bounded above by the square-root of the spectral norm of $\mathcal{A}$. Note that $\frac{1}{N}\sum_{i=1}^N \epsilon_i^2(\tilde{\mathbf{z}})$ is exactly the mean-square-residuals from a linear regression of the second-order-approximation on our linearized solution.

Proposition 6. *Let $\mathcal{A} : \mathbb{R}^N \to \mathbb{R}_{\geq 0}$ denote the order-4 symmetric tensor defined by the polynomial*

$$g(\tilde{\mathbf{z}}) = \frac{1}{N}\sum_{i=1}^N \left(\sum_{a,b,c,d=1}^N [\mathbf{H}_{f_i}]_{ab} \cdot [\mathbf{H}_{f_i}]_{cd} \cdot \tilde{z}_a \cdot \tilde{z}_b \cdot \tilde{z}_c \cdot \tilde{z}_d \right),$$

where $[\mathbf{H}_{f_i}]_{ab}$ is the ab-th entry of $\mathbf{H}_{f_i}$. By construction, $g(\tilde{\mathbf{z}}) = \langle \mathcal{A}, \tilde{\mathbf{z}} \otimes \tilde{\mathbf{z}} \otimes \tilde{\mathbf{z}} \otimes \tilde{\mathbf{z}} \rangle$ represents the inner product and is equal to the cross-equation sum-of-square of the second-order terms ($g(\tilde{\mathbf{z}}) = \frac{1}{N}\sum_i \epsilon_i^2(\tilde{\mathbf{z}})$) under productivity shock $\tilde{\mathbf{z}}$. Let $\mu^{\mathcal{A}}$ be the spectral norm of $\mathcal{A}$:

$$\mu^{\mathcal{A}} \equiv \sup_{\mathbf{z}} \frac{\langle \mathcal{A}, \mathbf{z} \otimes \mathbf{z} \otimes \mathbf{z} \otimes \mathbf{z} \rangle}{\|\mathbf{z}\|_2^4},$$

where $\|\cdot\|_2$ is the ℓ_2 norm ($\|\mathbf{z}\|_2 \equiv \sqrt{\mathbf{z}'\mathbf{z}}$). Then

$$\sqrt{\frac{1}{N}\sum_i \epsilon_i^2(\tilde{\mathbf{z}})} \leq \sqrt{\mu^{\mathcal{A}}}\|\tilde{\mathbf{z}}\|_2^2 = \sqrt{\mu^{\mathcal{A}}}\tilde{\mathbf{z}}'\tilde{\mathbf{z}}.$$

Proof. See Section B.6 of the online appendix. □

The spectral norm of $\mathcal{A}$ can be computed using the observed trade data, and the norm being close to zero implies that the second-order terms are close to zero. Furthermore, Lagrange's remainder theorem implies that if productivity shocks are bounded, we can obtain a bound on all the higher-order terms including second-order and above. Using $\mathbf{H}_{f_i}(\tilde{\mathbf{z}})$ to denote the Hessian of $f_i(\tilde{\mathbf{z}})$ evaluated at productivity shock $\tilde{\mathbf{z}}$ (not necessarily equal to the zero vector), we have the following result.

Proposition 7. *Suppose productivity shocks are bounded, $\tilde{\mathbf{z}} \in \mathcal{X} \equiv \prod_{i=1}^N [\underline{z}, \bar{z}]$. For any $\tilde{\mathbf{z}}$, there exists $\mathbf{x} \in \mathcal{X}$ such that*

$$\ln \hat{w}_i = \underbrace{-\theta(\ln \hat{w}_i - \tilde{z}_i) + \sum_n t_{in} \ln \hat{w}_n + \theta \sum_n m_{in} [\ln \hat{w}_n - \tilde{z}_n]}_{\text{first-order}} + \underbrace{\tilde{\mathbf{z}}' \mathbf{H}_{f_i}(\mathbf{x}) \tilde{\mathbf{z}}}_{\substack{\text{second and} \\ \text{higher-order}}}.$$

Proof. This is a direct application of Lagrange's remainder theorem. □

Proposition 7 demonstrates that the Hessian matrix, evaluated at some productivity shock vector $\mathbf{x}$, provides the *exact* error for our first-order approximation. A bound on the eigenvalue of the Hessian evaluated over the entire support $\mathcal{X}$ of productivity shocks therefore provides an upper-bound on the exact approximation error. We exploit this result in our empirical analysis below and show that approximation errors are close to zero for productivity shocks of the magnitude implied by the observed trade data. We thus conclude that our linearization provides an almost exact approximation to the full non-linear solution of the model given the observed trade matrices. Consistent with this, when we regress the full non-linear solution from the exact-hat algebra on our linear approximation in our empirical analysis below, we find R-squared close to one ($R^2 > 0.99$) in all of our simulations.

4 Extensions

We now consider a number of extensions to our friends-and-enemies measures of countries' income and welfare exposure to productivity shocks. In Section 4.1, we derive the corresponding matrix representations allowing for both productivity and trade cost shocks. In Section 4.2, we relax one of the ACR macro restrictions to allow for trade imbalance. In Section 4.3, we relax another of the ACR macro restrictions to consider small deviations from a constant elasticity import demand system. In Section 4.4, we show that our results generalize to a multi-sector model with a single constant trade elasticity following Costinot et al. (2012). In Section 4.5, we extend this specification further to introduce input-output linkages following Caliendo and Parro (2015). Finally, in Section 4.6, we show that our results also hold for economic geography models with factor mobility, including Helpman (1998), Redding and Sturm (2008), Allen and Arkolakis (2014), Ramondo et al. (2016) and Redding (2016).

4.1 Productivity and Trade Cost Shocks

Whereas productivity shocks are common across all trade partners, trade cost shocks are bilateral, which implies that our comparative static results in equations (13) and (14) now have a representation as a three tensor. To reduce this three tensor down to a matrix (two tensor) representation, we aggregate bilateral trade costs across partners using the appropriate weights implied by the model. In particular, we define two measures of outgoing and incoming trade costs, which are trade-share weighted averages of the bilateral trade costs across all export destination and import sources, respectively. We define *outgoing* trade costs for country i as $\mathrm{d}\ln\tau_i^{\mathbf{out}} \equiv \sum_n t_{in}\,\mathrm{d}\ln\tau_{ni}$, where the weights are the income share (t_{in}) that country i derives from selling to each export destination n. We define *incoming* trade costs for country n as $\mathrm{d}\ln\tau_n^{\mathbf{in}} \equiv \sum_i s_{ni}\,\mathrm{d}\ln\tau_{ni}$, where the weights are the expenditure share (s_{ni}) that country n devotes to each import source i. Using these definitions in equations (13) and (14), we obtain:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \theta \left(\begin{array}{c} \sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}\lambda_{nh}\left[\mathrm{d}\ln w_n - \mathrm{d}\ln z_n\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] \\ +\sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln\tau_n^{\mathbf{in}} - \mathrm{d}\ln\tau_i^{\mathbf{out}} \end{array} \right), \tag{29}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni}\left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] - \mathrm{d}\ln\tau^{\mathbf{in}}, \tag{30}$$

which enables us to obtain the following matrix representation.

Proposition 8. *Under ACR assumptions (i)-(iv) and macro restrictions (i)-(iii), the first-order general equilibrium impact of productivity and trade cost shocks on income and welfare in all countries around the world solves the following fixed point equations:*

$$\begin{aligned} \underbrace{\mathrm{d}\ln\mathbf{w}}_{\textit{income effect}} &= \underbrace{\mathbf{T}\,\mathrm{d}\ln\mathbf{w}}_{\textit{market-size effect}} + \underbrace{\theta\left[\mathbf{M}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right) + \mathbf{T}\,\mathrm{d}\ln\tau^{\mathbf{in}} - \mathrm{d}\ln\tau^{\mathbf{out}}\right]}_{\textit{cross-substitution effect}} \\ &= \mathbf{W}\,\mathrm{d}\ln\mathbf{z} + \theta\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{T}\,\mathrm{d}\ln\tau^{\mathbf{in}} - \mathrm{d}\ln\tau^{\mathbf{out}}\right) \end{aligned} \tag{31}$$

$$\underbrace{\mathrm{d}\ln\mathbf{u}}_{\textit{welfare effect}} = \underbrace{\mathrm{d}\ln\mathbf{w}}_{\textit{income effect}} \underbrace{-\mathbf{S}\left(\mathrm{d}\ln\mathbf{w}-\mathrm{d}\ln\mathbf{z}\right)+\mathrm{d}\ln\tau^{\mathbf{in}}}_{\textit{cost-of-living effect}} \tag{32}$$

Proof. The proposition follows from equations (29) and (30), as shown in Section F.1 of the online appendix. □

From Proposition 8, holding productivity constant, country n's demand for the goods supplied by country i increases if the bilateral trade cost τ_{ni} between these countries falls relative to country n's trade costs with all other nations. These effects are aggregated into $\mathrm{d}\ln\tau_n^{\mathbf{in}}$ and $\mathrm{d}\ln\tau_i^{\mathbf{out}}$, which weight the bilateral changes in trade costs by their appropriate income and expenditure shares. From equation (31), country i's income increases if its outgoing trade cost ($\mathrm{d}\ln\tau_i^{\mathbf{out}}$) falls relative to the incoming trade cost of its export markets, weighted by the importance of each market for country i's income ($\mathbf{T}\,\mathrm{d}\ln\tau^{\mathbf{in}}$). In this equation, productivity shocks are pre-multiplied by the matrix $\mathbf{M}$. In contrast, incoming trade cost shocks are pre-multiplied by the matrix $\mathbf{T}$, because they already include the expenditure share weights (s_{ni}), and outgoing trade cost shocks already incorporate the income share weights (t_{in}). From equation (32), incoming trade cost shocks ($\mathrm{d}\ln\tau^{\mathbf{in}}$) also directly affect welfare through a higher cost of imports, which raises the cost of living. In addition to these direct effects, trade cost shocks like productivity shocks also have indirect general equilibrium effects, through the resulting endogenous changes in incomes.

4.2 Trade Imbalance

Our bilateral friends-and-enemies exposure measures in equations (15) and (19) are derived under the ACR macro restrictions, including balanced trade. We now show that Propositions 1 and 2 naturally generalize to the case of exogenous trade imbalances commonly considered in the quantitative international trade literature. We measure the flow welfare of the representative agent as per capita expenditure deflated by the consumption price index:

$$u_n = \frac{w_n\ell_n + \bar{d}_n}{\ell_n\left[\sum_{i=1}^{N} p_{ni}^{-\theta}\right]^{-\frac{1}{\theta}}} \tag{33}$$

where $\bar{d}_n$ is the nominal trade deficit. Market clearing requires that income in each location equals expenditure on goods produced in that location:

$$w_i\ell_i = \sum_{n=1}^{N} s_{ni}\left[w_n\ell_n + \bar{d}_n\right]. \tag{34}$$

Trade Matrices We begin by establishing some properties our trade matrices under trade imbalance. We continue to use $q_i \equiv w_i\ell_i / \left(\sum_n w_n\ell_n\right)$ to denote country i's share of world income. Let $e_i \equiv \left(w_i\ell_i + \bar{d}_n\right) / \left(\sum_n w_n\ell_n\right)$ denote country i's share of world expenditures, where we use the fact that the aggregate deficit for the world as a whole is equal to zero. Let $d_i \equiv q_i/e_i$ denote country i's income-to-expenditure ratio, which is equal to one divided by one plus its nominal trade deficit relative to income. Let $\mathbf{D} \equiv Diag\left(\mathbf{d}\right)$ be the diagonalization of the vector $\mathbf{d}$; note $\mathbf{q}' = \mathbf{e}'\mathbf{D}$. Under trade balance, $q_i = e_i$ for all i, and $\mathbf{D} = \mathbf{I}$.

We continue to use $\mathbf{S}$ to denote the expenditure share matrix and $\mathbf{T}$ to denote the income share matrix: s_{ni} captures the expenditure share of importer n on exporter i and t_{in} captures the share of exporter i's income derived from selling to importer n. Under trade balance, $q_i t_{in} = q_n s_{ni}$, but this is no longer the case under trade imbalance. Instead, we have the following results.

Lemma 3. *Under trade imbalance,* $\mathbf{q}' = \mathbf{e}'\mathbf{S}$, $\mathbf{e}' = \mathbf{q}'\mathbf{T}$. *Moreover,*

1. $\mathbf{q}'$ *is the unique left-eigenvector of* $\mathbf{D}^{-1}\mathbf{S}$ *with all positive entries summing to one; the corresponding eigenvalue is one.* $\mathbf{q}'$ *is also the unique left-eigenvector of* $\mathbf{TD}$ *and* $\mathbf{TS}$ *with eigenvalue equal to one.*

2. $\mathbf{e}'$ *is the unique left-eigenvector of* $\mathbf{SD}^{-1}$ *with all positive entries summing to one; the corresponding eigenvalue is one.* $\mathbf{e}'$ *is also the unique left-eigenvector of* $\mathbf{DT}$ *and* $\mathbf{ST}$ *with eigenvalue equal to one.*

Proof. See Section B.1 of the online appendix. □

Comparative Statics Using these properties of the trade matrices, we now derive countries' income and welfare exposure to productivity shocks under trade imbalance. As the model does not generate predictions for how trade imbalances respond to shocks, we follow the common approach in the quantitative international trade literature of treating them as exogenous. In particular, we assume that trade imbalances are constant as a share of world GDP, which given our choice of world GDP as the numeraire, corresponds to holding the nominal trade deficits $\bar{d}_n$ fixed for all countries n.

Totally differentiating (33) and (34), we obtain the following generalizations of equations (13) and (14) to incorporate trade imbalances:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{ni}\left(\mathrm{d}\ln e_n + \theta\left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni}\right)\right), \tag{35}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln e_n - \sum_{m=1}^{N} s_{nm}\,\mathrm{d}\ln p_{nm}. \tag{36}$$

The introduction of trade imbalance has three main implications for these comparative static relationships. First, trade imbalances complicate the relationship between the expenditure share ($\mathbf{S}$), income share ($\mathbf{T}$) and cross-substitution ($\mathbf{M}$) matrices, because with income no longer equal to expenditure for each country ($e_i \neq q_i$), we have $q_i t_{in} \neq q_n s_{ni}$. Second, the market-size effect in the income equation depends on changes in expenditure rather than changes in income (the first term in equation (35)). Third, the income effect in the welfare equation also depends on changes in expenditure rather than changes in income (the first term in equation (36)). Under our assumption that trade imbalances stay constant as a share of world GDP, we have the following generalization of our earlier results.

Proposition 9. *Assume constant trade deficits* $\bar{d}_n$ *for all countries* n. *The general equilibrium impact of global productivity shocks on countries' income and welfare has the following bilateral "friends" and "enemies" matrix representations:*

$$\mathrm{d}\ln\mathbf{w} = \mathbf{W}\,\mathrm{d}\ln\mathbf{z}, \qquad \mathbf{W} \equiv -\frac{\theta}{\theta+1}\left(\mathbf{I} - \frac{\mathbf{TD}+\theta\mathbf{TS}}{\theta+1} + \mathbf{Q}\right)^{-1}\mathbf{M}, \tag{37}$$

$$\mathrm{d}\ln\mathbf{u} = \mathbf{U}\,\mathrm{d}\ln\mathbf{z}, \qquad \mathbf{U} \equiv (\mathbf{D}-\mathbf{S})\,\mathbf{W} + \mathbf{S}, \tag{38}$$

where recall that $\mathbf{D}$ *is the diagonalization of the vector of the ratio of income-to-expenditure* d_i.

Proof. The Proposition follows from equations (35) and (36), noting that for all n, $\mathrm{d}\ln\bar{d}_n = 0 \implies \mathrm{d}\ln e_n = \frac{q_n}{e_n}\,\mathrm{d}\ln w_n$, as shown in Section F.2 of the online appendix. □

4.3 Deviations from Constant Elasticity Import Demand

Our friends-and-enemies measures of countries' income and welfare exposure to small productivity shocks in equations (15) and (19) are only exact under the assumption of a constant elasticity import demand system. Using our characterization of the general Armington model in Section 2, we now examine the sensitivity of our exposure measures to deviations from this constant elasticity assumption. We begin by noting that a constant elasticity import demand system implies that the cross-price elasticities (θ_{nih}) in the market clearing condition (8) are:

$$\theta_{nih} = \begin{cases} (s_{nh} - 1)\,\theta & \text{if } i = h \\ s_{nh}\theta & \text{otherwise} \end{cases}. \tag{39}$$

Without loss of generality, we can represent the cross-price elasticity of any homothetic demand system as:

$$\theta_{nih} = \begin{cases} (s_{nh} - 1)\,\theta + o_{nih} & \text{if } i = h \\ s_{nh}\theta + o_{nih} & \text{otherwise,} \end{cases} \tag{40}$$

where o_{nih} captures the deviation from the predictions of the constant elasticity specification (39). Noting that homotheticity implies $\sum_{k=1}^{N} s_{nk} o_{nkh} = 0$, we obtain the following generalizations of our bilateral friend-enemy matrix representations of the income and welfare effects of productivity shocks:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\,\mathrm{d}\ln \mathbf{w} + (\theta \mathbf{M} + \mathbf{O}) \times (\,\mathrm{d}\ln \mathbf{w} - \,\mathrm{d}\ln \mathbf{z}\,)\,, \tag{41}$$

$$\mathrm{d}\ln \mathbf{U} = \,\mathrm{d}\ln \mathbf{w} - \mathbf{S}\,(\,\mathrm{d}\ln \mathbf{w} - \,\mathrm{d}\ln \mathbf{z}\,)\,, \tag{42}$$

where $\mathbf{O}$ is a matrix with entries $\mathbf{O}_{in} \equiv \sum_{h=1}^{N} t_{in} o_{nih}$ capturing the average across markets n of these deviations weighted by the share of country i's income derived from each market, as shown in Section F.3 of the online appendix. Using homotheticity, we can rewrite $\mathbf{O} \equiv \epsilon \cdot \bar{\mathbf{O}}$ as the product between a scalar $\epsilon > 0$ and a matrix $\bar{\mathbf{O}}$ with an induced 2-norm equal to one ($\|\bar{\mathbf{O}}\| = 1$). By construction, $\|\mathbf{O}\| = \epsilon$. Using this representation, we can use results from matrix perturbation to obtain an upper bound on the sensitivity of income exposure to departures from the constant elasticity model, as a function of the observed trade matrices and the trade elasticity.

Proposition 10. *Let $\widetilde{\mathrm{d}\ln \mathbf{w}}$ be the solution to the general Armington model in equation (8) and let $\mathrm{d}\ln \mathbf{w}$ be the solution to the constant elasticity of substitution (CES) Armington model in equation (15). Then*

$$\lim_{\epsilon \to 0} \frac{\|\,\widetilde{\mathrm{d}\ln \mathbf{w}} - \,\mathrm{d}\ln \mathbf{w}\|}{\epsilon \cdot \|\,\mathrm{d}\ln \mathbf{w}\|} \leq \frac{\theta}{\theta + 1} \|\,(\mathbf{I} - \mathbf{V})^{-1}\,\| \|\mathbf{I} - (\mathbf{W} + \mathbf{Q})^{-1}\|. \tag{43}$$

Proof. See Section B.7 of the online appendix. □

Given this upper bound on the sensitivity of income exposure from Proposition 10, we can use equation (42) to compute the corresponding upper bound on the sensitivity of welfare exposure. All terms on the right-hand side of equation (43) can be computed using the observed trade matrices and the trade elasticity. Therefore, we can can compute these upper bounds for alternative assumed values of the trade elasticity. An immediate corollary of Proposition 10 is that as the departures from the constant elasticity model become small ($\epsilon \to 0$), income exposure under a variable trade elasticity converges towards its value in our constant elasticity specification.

Corollary 3. *As the deviations from a constant elasticity import demand system become small ($\lim \epsilon \to 0$), income and welfare exposure in the general Armington model converge to their values in the constant elasticity of substitution (CES) Armington model.*

Proof. This corollary follows immediately from Proposition 10. □

From Corollary 3, we can interpret the constant elasticity model as a limiting case of the variable elasticity model. In the neighborhood of this limiting case, our friends-and-exposure income and welfare exposure measures approximate those for the variable elasticity model. More generally, from Proposition 10, we can provide an upper bound for sensitivity of income and welfare exposure to departures from the constant elasticity model that be computed using the observed trade matrices and assumed values for the trade elasticity.

4.4 Multiple Sectors

Our friends-and-enemies sufficient statistics extend naturally to a multi-sector model with a constant trade elasticity. For continuity of exposition, we focus on a multi-sector version of the constant elasticity Armington model from Section 3 above, but the same results hold in the multi-sector version of the Eaton and Kortum (2002) model following Costinot et al. (2012), as shown in Section F.4 of the online appendix. The preferences of the representative consumer in country n are now defined across the consumption of a number of sectors k according to a Cobb-Douglas functional form:

$$u_n = \frac{w_n}{\prod_{k=1}^{K}\left[\sum_{i=1}^{N}\left(p_{ni}^k\right)^{-\theta}\right]^{-\alpha_n^k/\theta}}, \qquad \sum_{k=1}^{K}\alpha^k = 1, \qquad \theta = \sigma - 1, \qquad \sigma > 1. \tag{44}$$

where $\sigma > 1$ is the elasticity of substitution between country varieties and $\theta = \sigma - 1$ is the trade elasticity.

Using expenditure minimization, the share of country n's expenditure in industry k on varieties from country i takes the standard constant elasticity form:

$$s_{ni}^k \equiv \frac{\left(p_{ni}^k\right)^{-\theta}}{\sum_{j=1}^{N}\left(p_{nj}^k\right)^{-\theta}}, \tag{45}$$

and we let $t_{in}^k \equiv s_{ni}^k \alpha_n^k \frac{w_n \ell_n}{w_i \ell_i}$ be the fraction of exporter i's income derived from selling to importer n in industry k.

Using the market clearing condition that country income equals expenditure on goods produced by that country, the impact of small changes in country productivity that are common across industries ($\mathrm{d}\ln z_\ell^k = \mathrm{d}\ln z_\ell$ for all k) on income and welfare in all countries has the following "friends" and "enemies" matrix representation:

$$\underbrace{\mathrm{d}\ln \mathbf{w}}_{\text{income effect}} = \underbrace{\mathbf{T}\,\mathrm{d}\ln \mathbf{w}}_{\text{market-size effect}} + \underbrace{\theta \mathbf{M}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right)}_{\text{cross-substitution effect}}, \tag{46}$$

$$\underbrace{\mathrm{d}\ln \mathbf{u}}_{\text{welfare effect}} = \underbrace{\mathrm{d}\ln \mathbf{w}}_{\text{income effect}} - \underbrace{\mathbf{S}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right)}_{\text{cost-of-living effect}}, \tag{47}$$

where the expenditure share matrix ($\mathbf{S}$), income share matrix ($\mathbf{T}$) and cross-substitution matrix ($\mathbf{M}$) are now:

$$\mathbf{S}_{ni} \equiv \sum_{k=1}^{K}\alpha_n^k s_{ni}^k, \qquad \mathbf{T}_{in} \equiv \sum_{k=1}^{K} t_{ni}^k = \sum_{k=1}^{K}\frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i}, \qquad \mathbf{M}_{in} \equiv \sum_{h=1}^{N}\sum_{k=1}^{K} t_{ih}^k s_{hn}^k - 1_{n=i}. \tag{48}$$

As in the single-sector model, $\mathbf{S}_{ni}$ equals the aggregate share of importer n's expenditure on goods produced by exporter i; $\mathbf{T}_{in}$ is again the aggregate share of exporter i's income derived from importer n; and $\mathbf{M}_{in}$ again captures the overall competitive exposure of country i to country n through each of their common markets (countries h and industries k), weighted by the importance of each market for i's income (t_{ih}^k).

Our income and welfare exposure measures in the multi-sector model again can be decomposed into the contribution of different economic mechanisms. From equation (46), productivity shocks affect income through the market-size effect, which is captured by the income share matrix $\mathbf{T}$, and the cross-substitution effect, which is captured by the matrix $\mathbf{M}$. Similarly, from equation (47), productivity shocks affect welfare through the income effect and the cost-of-living effect, where this cost-of-living effect depends on the expenditure share matrix $\mathbf{S}$. Both the income and welfare effect retain the decomposition into partial and general equilibrium effects using the series representation of the matrix inversion, as in equation (22) for the single-sector model above.

In this multi-sector model, changes in comparative advantage across industries provide an additional source of terms of trade effects between countries. Even common changes in productivity across all sectors have heterogeneous bilateral effects on income and welfare depending on the extent to which pairs of countries share similar patterns of comparative advantage across industries. Furthermore, we can examine the heterogeneous effects of these common changes in productivity across industries in trade partners using analogous sector-level measure of value-added exposure to global productivity shocks:

$$\mathrm{d}\ln \mathbf{Y}^k = \mathbf{W}^k \,\mathrm{d}\ln \mathbf{z}, \tag{49}$$

$$\mathbf{W}^k \equiv \mathbf{T}^k \mathbf{W} + \theta \mathbf{M}^k \left(\mathbf{W} - \mathbf{I} \right), \tag{50}$$

$$\mathbf{T}^k_{in} \equiv t^k_{ni}, \qquad \mathbf{M}^k_{in} \equiv \sum_{h=1}^{N} t^k_{ih} s^k_{hn} - 1_{n=i},$$

where $\mathbf{Y}^k$ is the vector of value-added in sector k across countries. Aggregating across sectors, our overall income exposure measure ($\mathbf{W}$) is the weighted average of these sector-level value-added exposure measures ($\mathbf{W^k}$), with weights equal to sector value-added shares:

$$\mathbf{W}_i = \sum_k r^k_i \mathbf{W}^k_i, \qquad r^k_i \equiv \frac{w_i \ell^k_i}{\sum_{h=1}^{K} w_i \ell^h_i}, \tag{51}$$

where $\mathbf{W}_i$ is the income exposure vector for country i with respect to productivity shocks in its trade partners n and $\mathbf{W}^k_i$ is the analogous sector value-added exposure vector for country i and sector k.

4.5 Multiple Sectors and Input-Output Linkages

We now show that we can further generalize this specification with multiple sectors from the previous subsection to incorporate input-output linkages, following Caliendo and Parro (2015). Again for continuity of exposition, we focus on a multi-sector version of the constant elasticity Armington model, but the same results hold in a multi-sector version of the Eaton and Kortum (2002) model, as in Caliendo and Parro (2015).

The representative consumer's preferences are again defined across the consumption of a number of sectors, as in equation (44) in the previous subsection. Within each sector, each country's good is produced with labor and composite intermediate inputs according to a constant returns to scale production technology. These goods are subject to iceberg trade costs, such that $\tau^k_{ni} \geq 1$ units must be shipped from country i to country n in sector k in order for one unit to arrive (where $\tau^k_{ni} > 1$ for $n \neq i$ and $\tau^k_{nn} = 1$). Therefore, the cost to a consumer in country n of purchasing a good from country i within sector k is:

$$p^k_{ni} = \tau^k_{ni} c^k_i, \qquad c^k_i = \left(\frac{w_i}{z^k_i} \right)^{\gamma^k_i} \prod_{j=1}^{K} \left(P^j_i \right)^{\gamma^{k,j}_i}, \qquad \sum_{k=1}^{K} \gamma^{k,j}_i = 1 - \gamma^k_i, \tag{52}$$

where c^k_i denotes the unit cost function for sector k and country i; γ^k_i is the share of labor in production costs in sector k in country i; $\gamma^{k,j}_i$ is the share of materials from sector j used in sector k in country i; and z^k_i captures value-added productivity in sector k in country i.

Country income and welfare exposure to global productivity shocks continue to have the "friends" and "enemies" matrix representation in equations (15) and (19). These exposure measures are again summarized by the expenditure share ($\mathbf{S}$), income share ($\mathbf{T}$) and cross-substitution ($\mathbf{M}$) matrices. As before, $\mathbf{S}_{ni}$ is the expenditure share of consumers in market n on the *value-added* of country i, $\mathbf{T}_{in}$ is the share of value-added that country i derives from country n, and $\mathbf{M}_{in}$ is the competitive exposure of country i to country n. However, the elements of these matrices now differ, because they use the observed input-output matrix to take into account the full structure of the network.

We now show how the elements of these matrices capture the network structure, with the full derivations reported in Section F.6 of the online appendix. We use i, n, h, o to index countries and j, k to index industries. The elements of the expenditure share matrix $\mathbf{S}_{ni}$ are now network adjusted as follows:

$$\mathbf{S}_{ni} \equiv \sum_{h=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{nh}^k \Lambda_{hi}^k, \tag{53}$$

where the first summation is across countries h and the second summation is across industries k; α_n^k is market n's Cobb-Douglas expenditure share for industry k; s_{nh}^k is the share of market n's expenditure within that industry on country h; Λ_{hi}^k captures the share of revenue in industry k in country h that is spent on value-added in country i. Similarly, the elements of the income share matrix $\mathbf{T}_{in}$ are now also network adjusted as follows:

$$\mathbf{T}_{in} \equiv \sum_{h=1}^{N} \sum_{k=1}^{K} \Pi_{ih}^k \vartheta_{hn}^k, \tag{54}$$

where the first summation is across countries h and the second summation is across industries k; Π_{ih}^k is the network-adjusted income share that country i derives from selling to industry k in country h; and ϑ_{hn}^k is the share of revenue that industry k in country h derives from selling to country n. Finally, the elements of the cross-substitution matrix are also network adjusted as follows:

$$\mathbf{M}_{in} \equiv \sum_{h=1}^{N} \sum_{k=1}^{K} \sum_{o=1}^{N} \Pi_{io}^k \left(\vartheta_{oh}^k + \sum_{j=1}^{N} \Theta_{oh}^{kj} \right) \Upsilon_{hon}^k, \tag{55}$$

where the first summation is across countries h, the second summation is across industries k, and the third summation is across countries o; Π_{io}^k is the network-adjusted share of income in country i derived from selling to country o in industry k; ϑ_{oh}^k is the share of revenue in industry k in country o that is derived from selling to country h; Θ_{oh}^{kj} captures the fraction of revenue in industry k in country o derived from selling to producers in industry j in country h; Υ_{noh}^k captures the responsiveness of country h's expenditure on industry k in country o with respect to a shock to costs in country n.

4.6 Economic Geography

Finally, we show that our constant elasticity Armington trade model in Section 3 can be generalized to incorporate labor mobility across locations, as in models of economic geography, including Helpman (1998), Redding and Sturm (2008), Allen and Arkolakis (2014), Ramondo et al. (2016) and Redding (2016). The economy as a whole is endowed with an exogenous measure of workers $\bar{\ell}$, each of whom has one unit of labor that is supplied inelastically. Workers are perfectly mobile across locations, but have idiosyncratic preferences for each location, which are drawn from an extreme value distribution.

As in the Armington trade model without labor mobility, we can use the market clearing condition that equates income in each location to expenditure on goods produced in that location to examine the impact of productivity shocks on income in all locations. Unlike the Armington trade model, population in each location in this market clearing condition is now endogenously determined by a population mobility condition. Using these market clearing and population mobility conditions, the impact of small productivity shocks on income in all locations again has a bilateral "friends" and "enemies" matrix representation:

$$\mathrm{d}\ln\mathbf{w} = \mathbf{T}\,\mathrm{d}\ln\mathbf{w} + \left[\left(\frac{(\sigma-1)-\kappa}{1+\kappa}\right)\mathbf{TS} - \left(\frac{\sigma-1}{1+\kappa}\right)\mathbf{I} + \frac{\kappa}{1+\kappa}\mathbf{S}\right](\,\mathrm{d}\ln\mathbf{w} - \,\mathrm{d}\ln\mathbf{z})\,, \tag{56}$$

as shown in Section F.7 of the online appendix. Having solved for this impact of the productivity shock on wages from equation (56), we can use these solutions in the population mobility condition to determine its impact of the population share of each location (ξ_n):

$$\mathrm{d}\ln\boldsymbol{\xi} = \kappa\,(\mathbf{I} - \boldsymbol{\Xi})\,[\,\mathrm{d}\ln\mathbf{w} - \mathbf{S}\,(\,\mathrm{d}\ln\mathbf{w} - \,\mathrm{d}\ln\mathbf{z})]\,, \tag{57}$$

where $\boldsymbol{\Xi}$ is a matrix in which each row is equal to vector of population shares across locations. Population mobility ensures that the impact of the productivity shock on expected utility (including the idiosyncratic preference shock) is equalized across all locations. Using our solutions for wages from equation (56) in the population mobility condition, we also can recover this impact on the common level of expected utility:

$$\mathrm{d}\ln\bar{u} = \boldsymbol{\xi}'\,[\,\mathrm{d}\ln\mathbf{w} - \mathbf{S}\,(\,\mathrm{d}\ln\mathbf{w} - \,\mathrm{d}\ln\mathbf{z})]\,, \tag{58}$$

where $\boldsymbol{\xi}$ is the vector of population shares of locations.

As in the trade model without labor mobility, income and welfare exposure to productivity shocks depend on the expenditure share matrix ($\mathbf{S}$), the income share matrix ($\mathbf{T}$) and the product of these two matrices that captures cross-substitution ($\mathbf{TS}$). In addition, in the economic geography model with labor mobility, both welfare exposure and the population response to these productivity shocks depend on population shares (though $\boldsymbol{\xi}$ and $\boldsymbol{\Xi}$).

5 Economic Friends and Enemies

In this section, we report our main empirical results for country income and welfare exposure to productivity shocks. In Subsection 5.1, we introduce our international trade data. In Subsection 5.2, we examine the quality of the approximation of our linearization to the full non-linear solution of the model for the empirical distribution of productivity shocks implied by the observed data. In Subsection 5.3, we use our baseline constant elasticity Armington model from Section 3 to examine global income and welfare exposure to productivity shocks for more than 140 countries over more than 40 years from 1970-2012. In Subsection 5.4, we compare the predictions of our baseline constant elasticity Armington model to those of our extensions to incorporate multiple sectors and input-output linkages.

5.1 Data

Our data on international trade are from the NBER World Trade Database, which reports values of bilateral trade between countries for around 1,500 4-digit Standard International Trade Classification (SITC) codes, as discussed further in Section H of the online appendix. The ultimate source for these data is the United Nations COMTRADE database and we use an updated version of the dataset from Feenstra et al. (2005) for the time period 1970-2012.[5] We augment these trade data with information on countries' gross domestic product (GDP), population and bilateral distances from the GEODIST and GRAVITY datasets from CEPII.[6] In specifications incorporating input-output linkages, we use a common input-output matrix for all countries from Caliendo and Parro (2015). We use these datasets to construct the $\mathbf{T}$, $\mathbf{M}$ and $\mathbf{S}$ matrices for our three specifications of the single-sector constant elasticity Armington model (Section 3), our multi-sector extension (Section 4.4) and our input-output extension (Section 4.5). In our single-sector model, our baseline sample consists of an balanced panel of 143 countries over the 43 years from 1970-2012. In our multi-sector models, we report results aggregating the products in the NBER World Trade Database to 20 International Standard Industrial Classification (ISIC) industries for which we have input-output data.

5.2 Quality of the Approximation and Computational Burden

We begin by comparing the predictions from our friend-enemy (first-order) linearization with those from the conventional exact-hat algebra approach. First, we undertake this comparison using the empirical distribution of productivity shocks implied by the observed trade data. Second, we report the results from broader comparisons using simulated productivity shocks. Third, we compare the computational performance of the two approaches.

Empirical Distribution of Productivity Shocks To compare our linearization with exact-hat algebra for empirically-reasonable productivity shocks, we begin by recovering the empirical distribution of productivity and trade cost shocks that rationalize the observed trade data in our baseline single-sector constant elasticity Armington model. Note that changes in productivity and trade costs are only separately identified up to a normalization or choice of units, because an increase in a country's productivity is isomorphic to a reduction in its trade costs with all partners (including itself). To separate these two variables, we use the normalization that there are no changes in own trade costs over time ($\hat{\tau}_{nn} = 1$), which absorbs common unobserved changes in trade costs across all partners into changes in productivity. But our findings for the quality of our approximation are not sensitive to the way in which we recover productivity shocks, as explored in the Monte Carlo simulations below.

We use this normalization and an assumed standard value of the trade elasticity of $\theta = 5$ to recover changes in trade costs and productivities ($\hat{\tau}_{ni}$, $\hat{z}_i$) from the model's gravity equation for bilateral trade flows and its market clearing condition that equates a country's income with expenditure on the goods produced by that country, as discussed further in Section G.1.1 of the online appendix. In Figure 1a, we display the empirical distribution of log changes in productivities ($\ln \hat{z}_i$) implied by the observed data from 2000-2010. As apparent from the figure, we find that these log changes in productivities are clustered relatively closely around their mean of zero, although some individual countries can experience large changes in log productivities, in part because any common trade cost shocks across all partners are absorbed into these changes in log productivities.

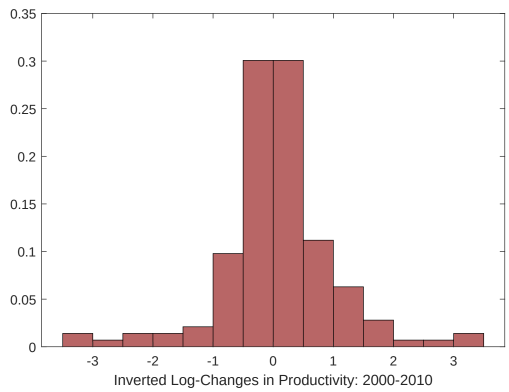
Having recovered these changes in productivities ($\hat{z}_i$) implied by the observed trade data, we now compare the predictions from our (first-order) linearization for the impact of these productivity shocks on income to those from the non-linear exact-hat algebra approach in equation (26). In particular, we set countries' productivity shocks equal to their empirical values ($\hat{z}_i$), undertake an exact-hat algebra counterfactual holding trade costs constant ($\hat{\tau}_{ni} = 1$), and

[5] See https://cid.econ.ucdavis.edu/wix.html.
[6] See http://www.cepii.fr/cepii/en/bdd_modele/bdd.asp.

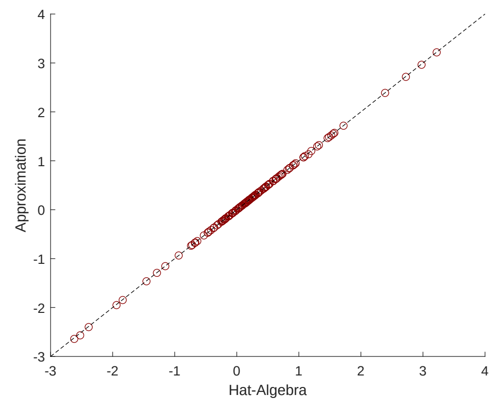
solve for the counterfactual changes in countries' per capita incomes ($\hat{w}_i$). We compare the results from these exact-hat algebra counterfactuals to the predictions of our linearization, which implies a log change in countries' per capita incomes in response to these productivity shocks of $\ln \hat{\mathbf{w}} = \mathbf{W} \ln \hat{\mathbf{z}}$. We also undertake an analogous exercise for changes in bilateral trade costs ($\hat{\tau}_{ni}^{-\theta}$), in which we undertake counterfactuals holding productivities constant ($\hat{z}_i = 1$), and compare the counterfactual changes in countries' per capita incomes from the exact-hat algebra counterfactuals ($\hat{w}_i$) to the predictions of our linearization, as discussed in Section G.1.1 of the online appendix.

Figure 1: Empirical Distribution of Productivity Shocks and Counterfactual Income Predictions

(a) Distribution Across Countries of Log Productivity Shocks ($\ln \hat{z}_{it}$) from 2000-2010



(b) Our Approximation Versus Exact-hat Counterfactual Predicted Changes in Income ($\ln \hat{w}_{it}$) from 2000-2010



Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3.

In Figure 1b, we display the predicted log changes in per capita incomes as a result of productivity shocks from our linearization and the exact-hat algebra counterfactuals. Although the two sets of predictions are not exactly the same as one another, we find an extremely strong relationship between them, such that they are almost visibly indistinguishable, with a correlation coefficient of more than 0.999. From Section 3.6 above, our approximation is exact under autarky ($s_{nn} \to 1$ and $t_{ii} \to 1$) and under free trade, and performs well using matrices of random trade shares scaled to sum to one. Empirically, we find that observed trade matrices are well approximated by a weighted average of autarky, free trade and random matrices, and hence our approximation also performs well using observed trade matrices. In Section G.1.1 of the online appendix, we report an analogous comparison for bilateral trade cost shocks. Although we again find a strong relationship between the predictions of our linearization and the exact-hat algebra counterfactuals, it is weaker than for productivity shocks. An important reason for this difference is that productivity shocks are common across all trade partners, which means that the direct effect of these productivity shocks can be taken outside of the summation across trade partners into a separate first term that is identical in our linearization and the exact hat algebra in equations (26) and (27). In contrast, the direct effect of bilateral changes in trade costs cannot be taken outside of this summation sign, because it varies across trade partners.

Simulated Productivity Shocks To explore the robustness of these results, we next report a broader set of comparisons between our linearization and the full non-linear solution of the model using simulated productivity shocks. In particular, we undertake 1,000 Monte Carlo simulations in which we draw (with replacement) productivity shocks for each country from the empirical distribution of productivity shocks from 2000-2010. Using these simulated productivity shocks, we undertake exact-hat algebra counterfactuals to compute predicted log changes in per capita income, and compare these predictions with those from our linearization. In Figure 3, we show the distribution of regression slope coefficients and R-squared between the two sets of predictions. Across all of our simulations, we find slope coefficients from 0.99-1.01 and correlation coefficients of more than 0.999.

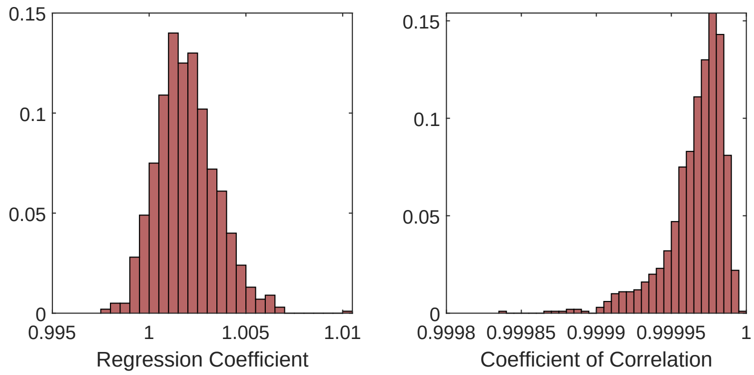
As a further robustness check, we multiplied the size of the productivity shocks by 1,000, and undertook another 1,000 Monte Carlo simulations. Even with productivity shocks three orders of magnitude larger than those implied by the observed trade data, we continue to find the same pattern of results, with a correlation coefficient of above 0.99 in all of our simulations. To explore the sensitivity of these results with respect to our assumed trade elasticity, we experimented with values for the trade elasticity from 2 to 20, which spans the empirically-relevant range of values for this parameter. Even for trade elasticities as extreme as 2 and 20, we continue to find regression slope coefficients ranging from 0.85-1.10 and correlation coefficients of above 0.99, as reported in Section G.1.1 of the online appendix. Taken together, these results suggest that our friend-enemy income exposure measures for productivity shocks are close to exact for empirically-reasonable changes in productivities and values of trade elasticities.

Bounds on Approximation Error Further light on these empirical results comes from our analytical results for the quality of the approximation in Propositions 3-7 in Subsection 3.6 above. In Table G.1 in Section G.1.1 of the Online Appendix, we report the distribution of Hessian eigenvalues over our sample period. We find that even the largest eigenvalue of the Hessian matrix ($\mathbf{H}_{f_i}$) is close to zero for each country. Therefore, as we approximate the log-income change for each country i separately, the second-order term ϵ_i, when maximized by a country-specific vector of TFP shocks $\left\{\tilde{\mathbf{z}}^{\max,i}\right\}$, accounts for at most a tiny fraction of the variation in $\ln\hat{w}_i$. For example, for the year 2000 and on average over time, we find that the second-order approximation error for the income exposure of each country is bounded by 0.26 percent and 0.36 percent of the variance of productivity shocks respectively.

Furthermore, for all countries, we find that the second-largest eigenvalues μ_i^{2nd} are substantially closer to zero, which implies that any productivity shock vector that is orthogonal to $\tilde{\mathbf{z}}^{\max,i}$ generates approximately zero second-order effects. We further find that the standard unit vector $\mathbf{e}^{\ell}$ comes close to achieving the upper bound for the ℓ-th equation, i.e. $\mathbf{e}^{\ell}\approx\tilde{\mathbf{z}}^{\max,\ell}$ for all ℓ. Hence, the second-order term for evaluating the effect of a productivity shock in country ℓ on income in country $i\neq\ell$ is small (approximately bounded by $\left|\mu^{2nd,i}\right|$) even relative to the own-effect on country ℓ itself, which is already small (approximately bounded by $\left|\mu^{\max,i}\right|$). Even considering all higher-order terms together (second-order and above) in Proposition 7, and using the assumption that the Hessian eigenvalues evaluated over the support of the distribution of productivity shocks are bounded by the Hessian eigenvalues observed during our sample period, we continue to find that the approximation error remains small. In particular, we find that the global approximation errors for income exposure to own productivity shocks are less than 0.62 percent of the variance of productivity shocks, and that these global approximation errors for income exposure to other countries' productivity shocks are 0.33 percent of the variance of productivity shocks.

Computational Speed In comparing our (first-order) linearization to the exact-hat algebra, another relevant criterion alongside the quality of the approximation is the relative computational burden. We compare computational speed for the two approaches using Matlab, a single thread (virtual CPU core), and a tolerance of 10^{-6} for solving the full non-linear solution of the model using the exact-hat algebra (our matrix inversion uses machine precision). We compute 6,149 comparative statics for country productivity shocks (143 countries $\times$ 43 years) for our baseline

Figure 2: Distributions of Regression Slope Coefficients and Coefficients of Correlation Comparing our Friend-Enemy Approximation to Exact-Hat Algebra Predictions in Monte Carlos using Simulated Productivity Shocks (Trade Elasticity $\theta = 5$)



Source: NBER World Trade Database and authors' calculations authors' calculations using our baseline constant elasticity Armington model from Section 3. Monte Carlo simulations using 1,000 replications. Simulated productivity shocks drawn (with replacement) from the empirical distribution of productivity shocks from 2000-10.

single-sector Armington model from Section 3. On both our laptops and high-performance computer servers, we find that our linearization is around 70,000 times faster than the exact-hat algebra.[7] As we move from our baseline single-sector specification to the more computationally demanding input-output specification, we find that this difference in processing time increases further. As a result of these improvements in computational speed, it becomes feasible to compare the results of large numbers of counterfactuals across different quantitative models, such as our single-sector, multi-sector and input-output specifications. Therefore, in settings in which large numbers of counterfactuals must be undertaken, our linearization can provide a useful complement to solving for the full non-linear solution using exact-hat algebra. At the very least, using the predictions of our linearization as the initial guess for the full model solution brings dramatic improvements in computational speed. More broadly, our approach closely approximates the full model solution, has a clear interpretation in terms of the underlying economic mechanisms, and enables researchers to easily explore the sensitivity of counterfactual predictions across different quantitative models.

5.3 Aggregate Income and Welfare Exposure 1970-2012

We now present our main empirical results on global income and welfare exposure to productivity shocks using our baseline constant elasticity Armington model from Section 3. First, we present results for the overall distribution of income and welfare exposure across countries and over time. Second, we provide further evidence on the large-scale changes in bilateral networks of income and welfare exposure that have occurred over our sample period. Third, we evaluate the role of general equilibrium relative to partial equilibrium effects in shaping the impact of these productivity shocks. Fourth, we examine the different economic mechanisms of the market-size, cross-substitution and cost-of-living effects. Fifth, we investigate the contributions of importer, exporter and third market effects in shaping countries' exposure to foreign productivity shocks.

[7] To solve the exact-hat algebra counterfactuals, we use an iterative algorithm of the form considered in Allen and Arkolakis (2014) and Allen et al. (2020). Using a standard Matlab optimization routine such as Fsolve substantially increases the computation time for these counterfactuals. Allowing for parallelization (multiple virtual CPU cores) reduces this computation time.

5.3.1 Global Income and Welfare Exposure

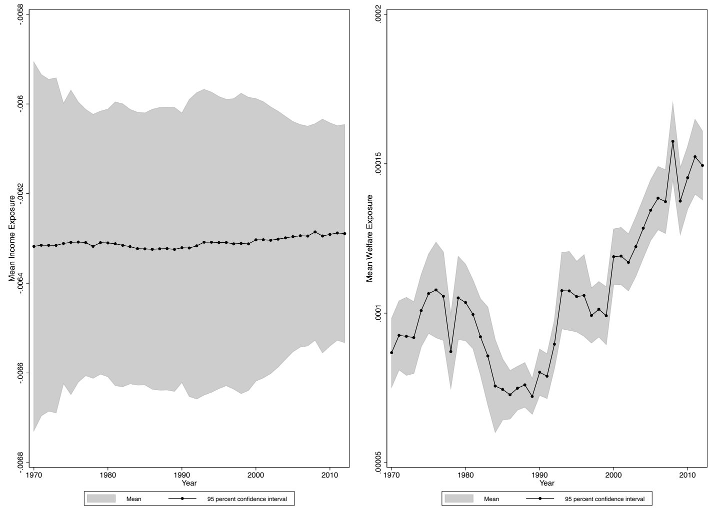
Using our baseline constant elasticity Armington model, we compute bilateral income and welfare exposure to productivity shocks for our balanced panel of 143 countries over the 43 years from 1970-2012 ($143 \times 143 \times 43 = 879,307$ bilateral predictions for each variable). In Figure 3, we show mean income and welfare exposure to foreign productivity shocks over time (excluding own productivity shocks) and their 95 percent confidence intervals. In interpreting the magnitudes, note that these values correspond to mean changes in income and welfare with respect to infinitesimally small productivity shocks. Once we take into account the empirical size of productivity shocks, we obtain predicted changes in income and welfare comparable to those from the full non-linear model solution, as shown in Section 5.2 above. Given our choice of world GDP as numeraire, a productivity shock that raises a country's own per capita income tends to reduce the per capita income of other countries (in order to hold world GDP constant), which results in a negative average income exposure (left panel). As our choice of numeraire holds world GDP constant over time, we also find that mean income exposure is relatively flat over time. Besides raising a country's own per capita income, a productivity shock also reduces its prices, and we find that this cost of living effect is sufficiently strong that average welfare exposure is positive (right panel). These welfare results are invariant to the choice of numeraire, which cancels from the income and cost of living components of welfare, as discussed above. One striking feature of the figure is the substantial and statistically significant increase in average welfare exposure over time, by around 72 percent from 1970-2012. This pattern of results is consistent with the view that the increased globalization that occurred over our sample period enhanced countries' interdependence on one another, as captured by average exposure to foreign productivity growth.

Another striking feature of Figure 3 is the substantial dispersion in exposure to foreign productivity shocks, as reflected in the 95 percent confidence intervals. In Figure 4, we provide further evidence on this heterogeneity in welfare exposure using Box and Whisker plots, in which the interquartile range is shown by the edges of the box, and the extended lines represent the 5th and 95th percentiles. Although on average foreign productivity shocks raise importer welfare, an importer at the 5th percentile experiences a reduction in welfare only somewhat smaller than the increase in welfare enjoyed by an importer at the 95th percentile. Furthermore, we find an increase in the dispersion of welfare exposure from the early 1980s onwards in Figure 4, which suggests that increased globalization has not only raised countries average exposure to foreign productivity growth, but also enhanced the inequality in the effects of this productivity growth, namely the extent to which individual countries are winners or losers from expansions in the productive capacity of particular trade partners.[8]

In our constant elasticity Armington model, this heterogeneity in welfare exposure reflects the interaction of the cross-substitution, market-size and cost-of-living effects. First, the direct effect of a country's productivity growth in lowering its prices has a negative cross-substitution effect on the per capita income of its trade partners, as these trade partners face increased competition in markets around the world. Second, this direct effect of productivity growth in lowering prices also raises welfare in all countries through a lower cost of living. Third, productivity growth raises a country's own per capita income, which has a positive market-size effect on the per capita income of other countries. Fourth, the resulting endogenous changes in per capita income in all countries have further indirect effects on prices, income and welfare through these cross-substitution, market-size and cost-of-living mechanisms. The relative balance

[8]The step increase in the dispersion of welfare exposure between 1999 and 2000 in Figure 4 is driven by a step increase in the number of bilateral importer-exporter pairs with positive international trade flows between those years.

Figure 3: Mean Income and Welfare Exposure to Productivity Shocks in Other Countries over Time



Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3.

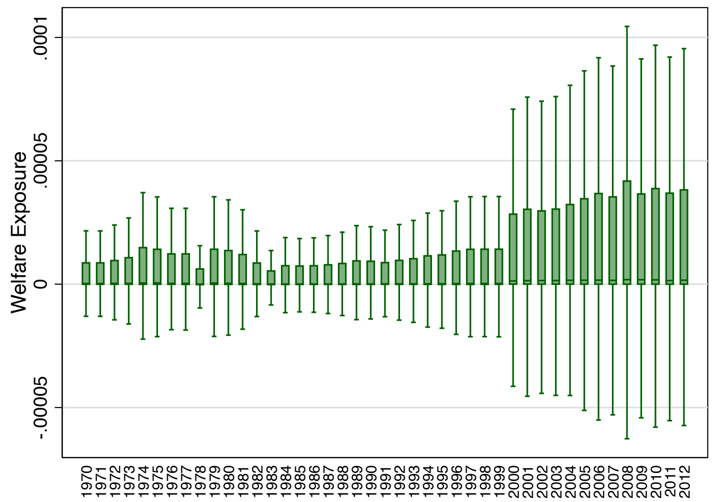
of all of these forces depends on the geography of trade flows, as reflected in the expenditure share matrix ($\mathbf{S}$), the income share matrix ($\mathbf{T}$), and the cross-substitution matrix ($\mathbf{M}$).

In Figure 5, we illustrate global welfare exposure to productivity shocks in 1970, 1985, 2000 and 2012 using a network graph, where the nodes are countries and the edges capture bilateral welfare exposure. For legibility, we display the 50 largest countries in terms of GDP and the 200 edges with the largest absolute values of bilateral welfare exposure.[9] The size of each node captures the importance of each country as a source of productivity shocks (as a source of welfare exposure for other countries); the arrow for each edge shows the direction of bilateral welfare exposure (from the source of the productivity shock to the exposed country); and the thickness of each edge shows the absolute magnitude of the bilateral welfare exposure. Countries are grouped to maximize modularity (the fraction of edges within the groups minus the expected fraction if the edges were distributed at random).

At the beginning of our sample period in 1970, the global network of welfare exposure is dominated by the U.S., Germany and other Western industrialized countries (top-left panel). Moving forward to 1985, we see the emergence of Japan and a cluster of Newly Industrialized Countries (NICs) in Asia, and we observe Western Europe increasingly emerging as a separate cluster of interdependent nations. By the time we reach 2000, the separate clusters of countries in Asia and Western Europe become even more apparent, with China beginning to displace Japan at the center of the Asian cluster. By the end of our sample period in 2012, China replaces the U.S. at the center of the global network of welfare exposure, with the US more tightly connected to China and other Asian countries than to the cluster of Western European countries. Therefore, we find substantial changes, not only in the mean and dispersion of welfare exposure, but also in the network of bilateral interdependencies between countries.

[9] All of the bilateral welfare exposure links shown in the figure are positive.

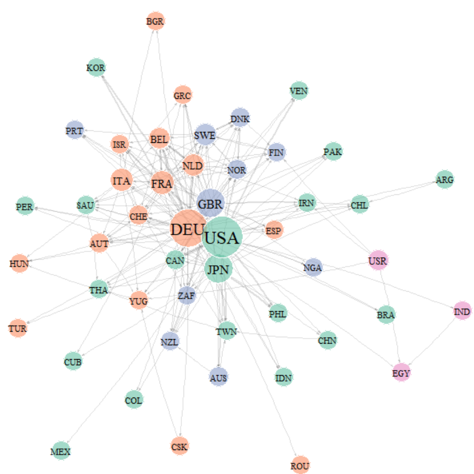
Figure 4: Box-Whisker Plot of Distribution of Welfare Exposure over Time



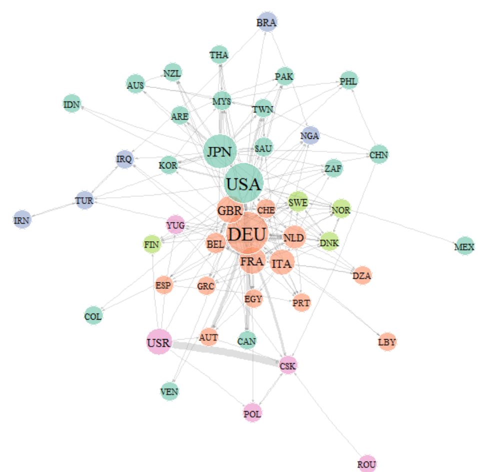
Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3; box shows interquartile range; extended lines show 5th and 95th percentiles.

Figure 5: Global Welfare Exposure, 1970, 1985, 2000 and 2012

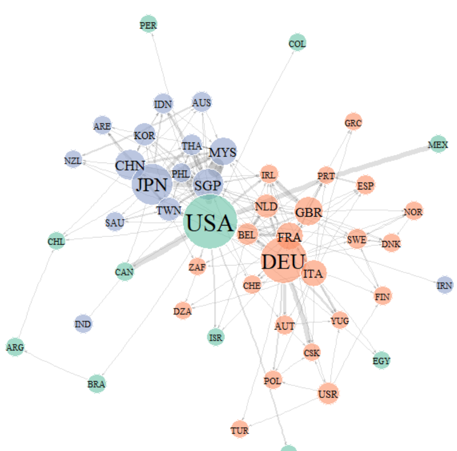
(a) 1970



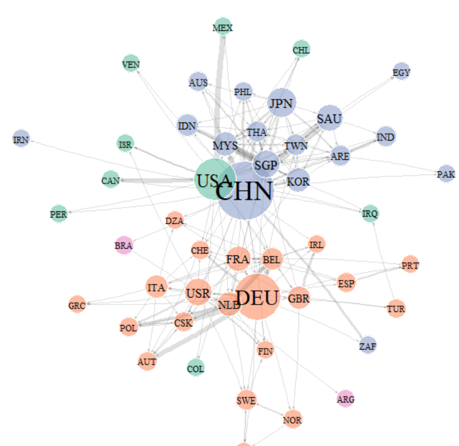
(b) 1985



(c) 2000



(d) 2012



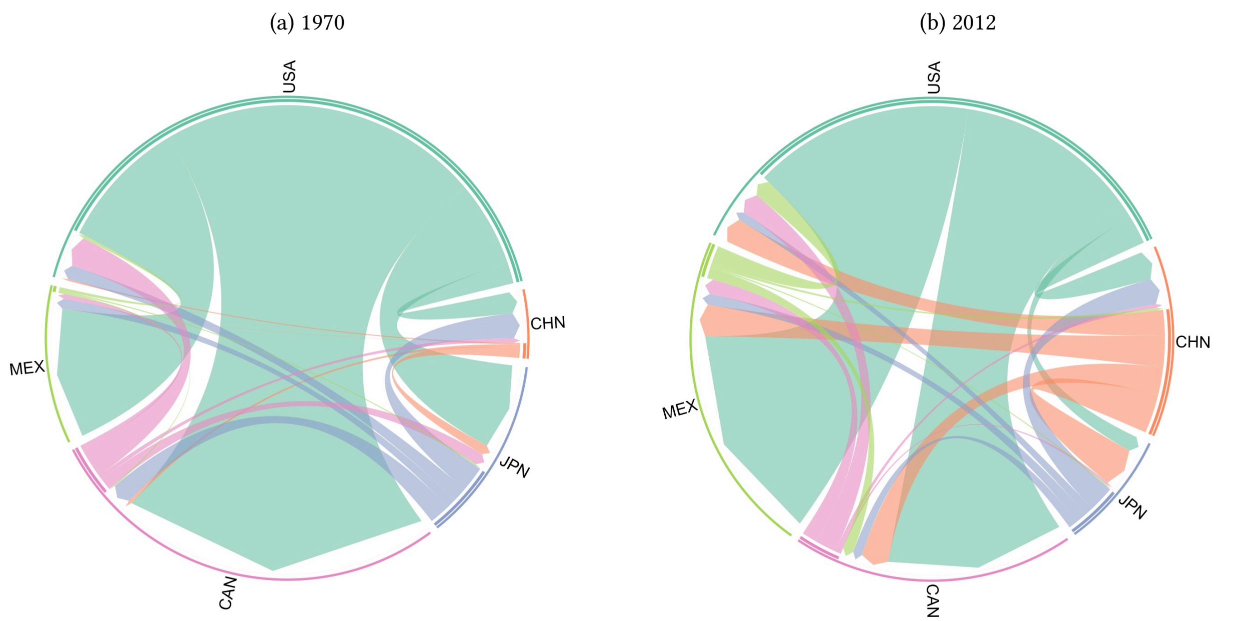
Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3.

5.3.2 Regional Networks of Welfare Exposure

To provide further evidence on changes in regional networks of welfare exposure over our sample period, we use chord or radial network diagrams, as used for example in comparative genomics in Krzywinski et al. (2009) and for bilateral migration flows in Sander et al. (2014).

In Figure 6, we show welfare exposure in 1970 and 2012 for U.S., Canada, Mexico, Japan and China, where each country is labelled by its three-letter International Organization for Standardization (ISO) code.[10] These countries are arranged around a circle, where the size of the inner segment for each country shows its overall outward exposure (the effect of its productivity shocks on other countries), and the gap between the inner and outer segments shows its overall inward exposure (the effect of foreign productivity shocks upon it). Arrows emerging from the inner segment for each country show the bilateral impact of its productivity shocks on welfare in other countries. Arrows pointing towards the gap between the inner and outer segments show the bilateral impact of other countries' productivity growth on its welfare.[11] In 1970, the network is dominated by the effect of US productivity shocks on welfare in the other countries, and Japan is substantially more connected to the network than China. By 2012, following Mexican trade liberalization in 1987, the Canada-US Free Trade Agreement (CUSFTA) in 1988 and the North American Free Trade Agreement (NAFTA) in 1994, we observe much deeper integration between the three North American economies. Additionally, we find a reversal of the relative positions of the two Asian economies, with China substantially more integrated into the network than Japan.

Figure 6: North American Welfare Exposure, 1970 and 2012



Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3.

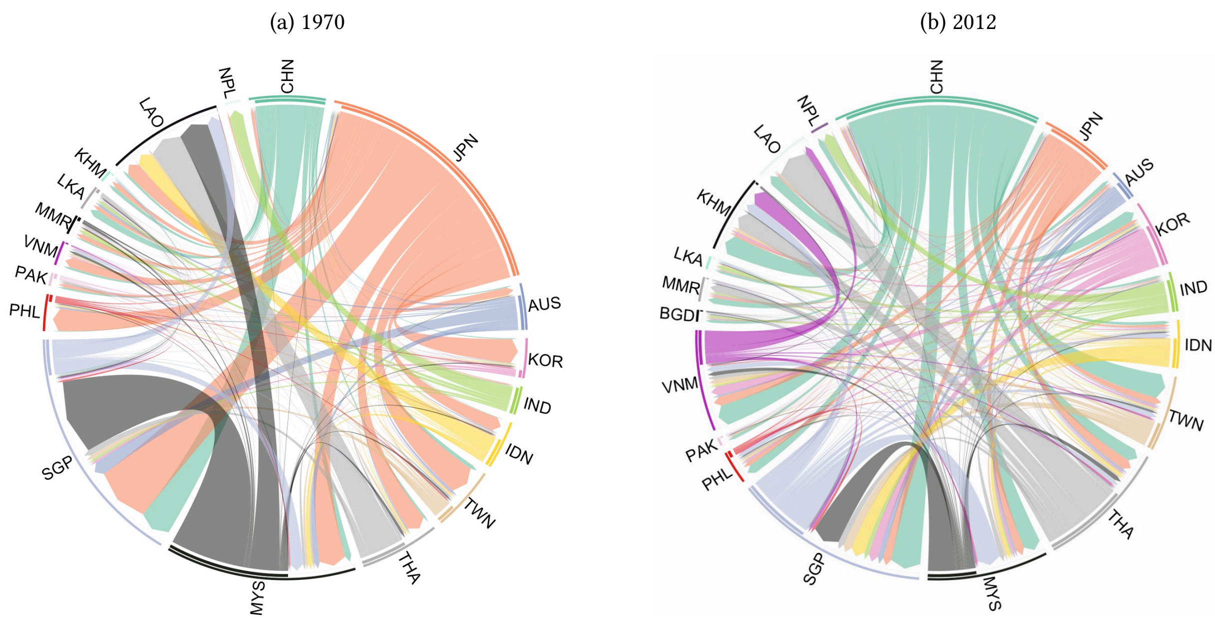
In Figure 7, we display welfare exposure for a broader group of Asian countries. Three features stand out. First, we again find a dramatic change in the relative positions of Japan and China. Whereas in 1970 Japan dominated

[10] To ensure a consistent treatment of countries over time, we manually assign some three-letter codes, such as the code USR for the members of the former Soviet Union.

[11] We omit own exposure to focus on the impact of foreign productivity shocks on country welfare. Almost all values of our welfare exposure measure in these diagrams are positive. For ease of interpretation, we add a constant to our welfare exposure measure in each year, such that its minimum value is zero, which implies that these diagrams show the impact of the productivity shock on relative levels of welfare.

the network of welfare exposure, in 2012 this position is firmly occupied by China. Second, Vietnam becomes both substantially more exposed to foreign productivity shocks and a much more important source of these productivity shocks for other countries, following its trade liberalization. Third, the overall network of welfare exposure is much denser in 2012 than in 1970, consistent with greater trade integration among these Asian countries increasing their economic interdependence on one another. In Section G.1.2 of the online appendix, we provide further evidence on large-scale changes in regional networks of welfare exposure for Central Europe following the fall of the Iron Curtain.

Figure 7: Asian Welfare Exposure, 1970 and 2012



Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3.

5.3.3 Partial and General Equilibrium Effects

We now examine the role of partial and general equilibrium forces in the model using our series decomposition in equation (22) above. From our earlier discussion, the direct or partial equilibrium effect of productivity growth in a given exporter is to increase its price competitiveness in all markets, which leads to substitution away from all other countries' goods. But there are also indirect or general equilibrium effects, as the endogenous changes in per capita income that occur in response to this productivity growth also affect both cross-substitution and market demand.

In Figures 8a and 8b, we show this series decomposition for the impact of Chinese productivity growth on U.S. income and welfare respectively. In both figures, the thick blue line shows the partial equilibrium effect (the first-order term $\frac{\theta}{\theta+1}\mathbf{M}$ in the series-decomposition in equation (22)). The thinner blue line immediately below adds to this partial equilibrium effect the first term from the general equilibrium component of the series decomposition (the term $\frac{\theta}{\theta+1}\mathbf{MV}$ in equation (22)). Each of the additional thinner blue lines further below add successively higher-order terms from the general equilibrium component of the series decomposition. As we add these additional higher-order terms, predicted income exposure converges towards our overall income exposure measure in equation (17).

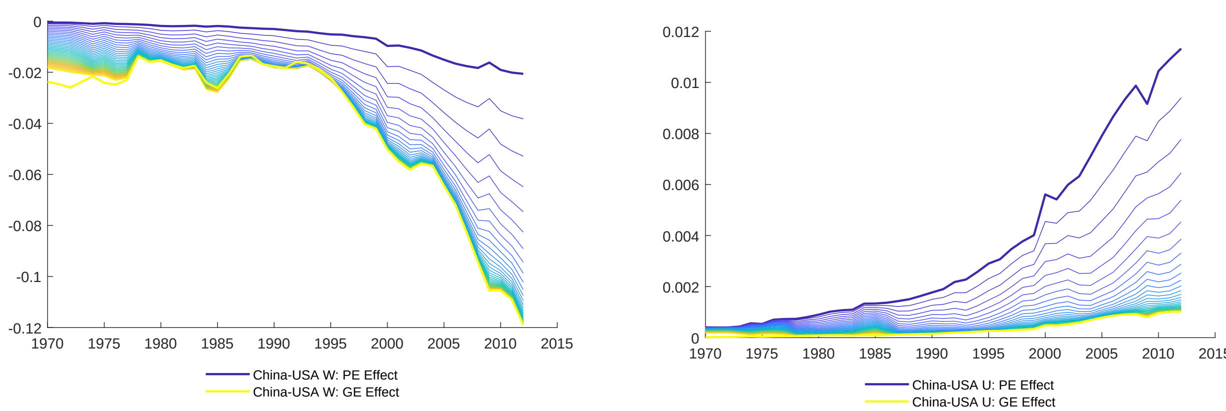
As apparent from the figure, we find that the general equilibrium forces in the model are large relative to the partial equilibrium forces, and we find relatively rapid convergence, such that the addition of a few higher-term terms in the series decomposition takes us relatively close to our overall measure of income exposure. Taken together, these results highlight the importance of general equilibrium forces in this class of constant elasticity trade models, and suggest that a misleading picture about the effects of productivity growth would be obtained by focusing solely on the partial equilibrium term of productivity growth weighted by the trade elasticity and the initial expenditure shares.

5.3.4 Cross-Substitution, Market Size and Cost of Living Effects

We now examine the contributions of the different economic mechanisms in the model by separating out overall income exposure into the contributions of the market-size and cross-substitution effects, and breaking down the welfare effect into the contributions of the income and cost of living effects.

Figure 8: Partial and General Equilibrium Effects of the Impact of Productivity Growth in China (Exporter) on Income in the United States (Importer) over Time

(a) Income Exposure (b) Welfare Exposure



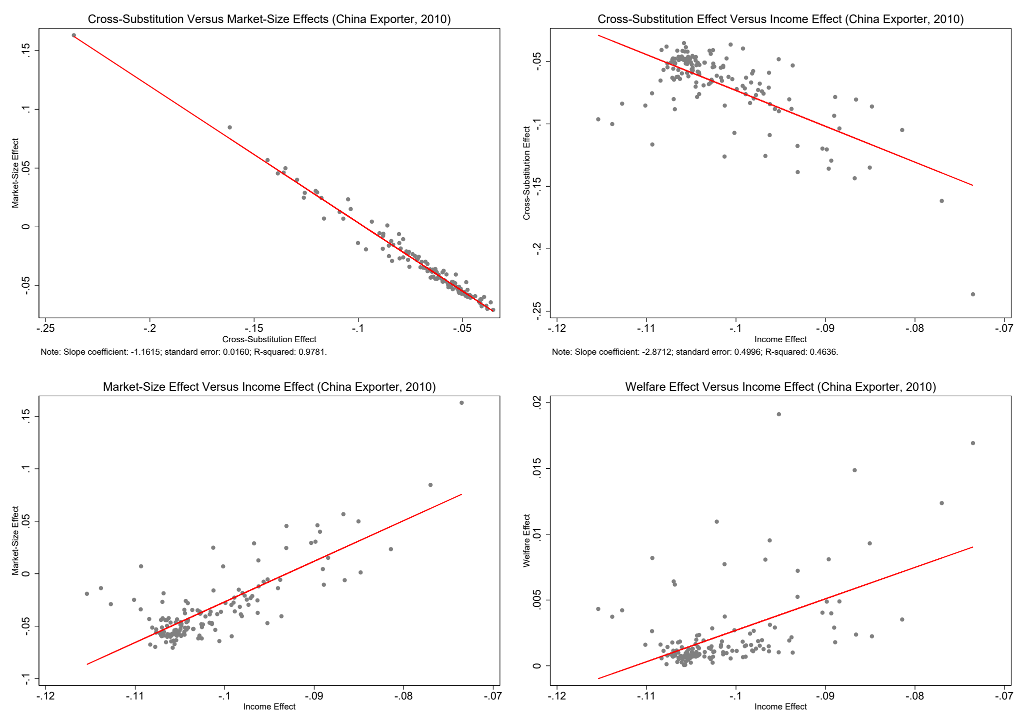
Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3.

In Figure 9, we illustrate these relationships for a Chinese productivity shock in 2010, where the circles correspond to each of the other countries in our sample (excluding China). In the top-left panel, we show the relationship between the cross-substitution effect ($\mathbf{W}^{\text{Sub}}$) and the market size effect ($\mathbf{W} - \mathbf{W}^{\text{Sub}}$). We find that the cross-substitution effect is always negative, as higher Chinese productivity reduces the competitiveness of other countries in all markets, which leads consumers in all markets to substitute away from these other countries' goods, and lowers their per capita income relative to China. In contrast, the market-size effect can be either positive or negative. On the one hand, higher productivity in China raises its per capita income, which increases the market demand for other countries' goods, and increases their income. On the other hand, the reduction in income in other markets from the cross-substitution effect lowers market demand for other countries' goods, which decreases their income. The overall income effect is the net outcome of these cross-substitution and market-size forces and hence can be either positive or negative. As apparent from the top-left panel, we find a strong relationship between the market-size and cross-substitution effects, because the gravity structure of international trade jointly determines the share of exporter i in importer n's expenditure (s_{ni}) and the share of importer n in exporter i's income (t_{in}), which are the key determinants of the relative magnitude of these two effects.

In the top-right panel, we display the cross-substitution effect against the overall income effect, while in the bottom-left panel, we show the market-size effect against the overall income effect. As the cross-substitution effect lowers per capita income, while the market size effect increases per capita income, we find a negative relationship in the top-right panel and positive relationship in the bottom left panel. Both the cross-substitution and market-size effects are quantitatively relevant relative to the overall income income, with a regression slope coefficient of -2.87 and R-squared of 0.46 in the top-right panel, and a regression slope coefficient of 3.87 and R-squared of 0.61 in the bottom-left panel. As we consider productivity shocks for different exporters and years, we find that exporter country size plays a central role in driving the relative importance of the market-size and cross-substitution effects in the overall income effect. In particular, the market size effect is smaller relative to the overall income effect for exporters with smaller shares of world GDP.

In the bottom-right panel, we show the welfare effect against the income effect, where these two effects differ from one another through the cost-of-living effect. As apparent from this panel, we find a positive and statistically significant relationship between the two variables, with a regression slope coefficient of 0.24. However, we find that this correlation is far from perfect, with a regression R-squared of only 0.28. This pattern of results highlights the strength of the cost-of-living effect in the model and emphasizes that caution is warranted in making inferences about changes in welfare from information on changes in income alone.

Figure 9: Mechanisms Underlying the Income and Welfare Effects



Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3.

5.3.5 Own and Third Market Effects

Even within the cross-substitution effect, our approach highlights that higher exporter productivity growth affects importer income and welfare through multiple channels of increased exporter price competitiveness in the importer's market, the exporter's market and third markets.

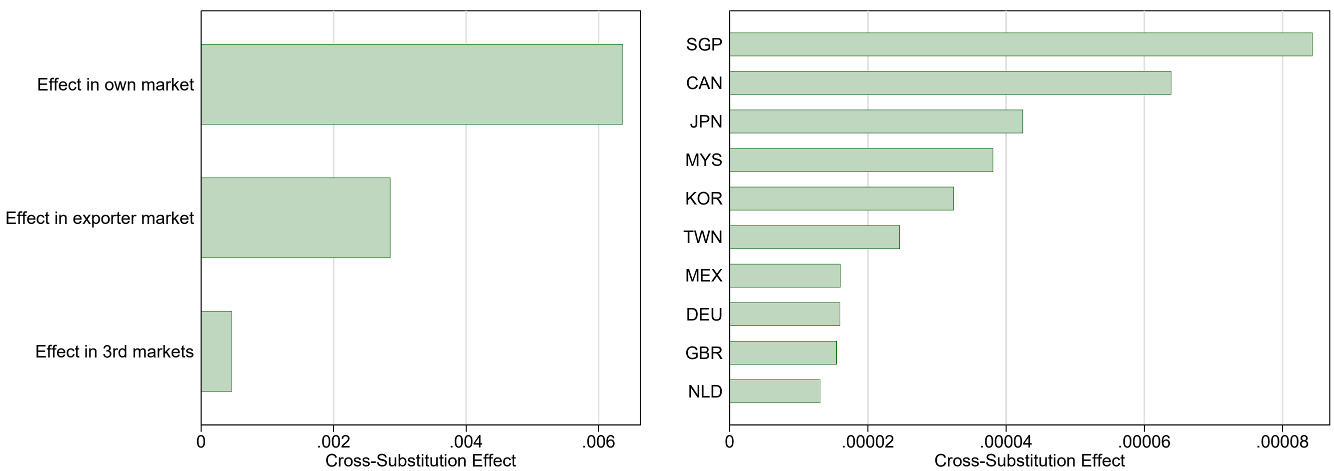
In Figure 10, we illustrate the contributions of these different terms towards the partial equilibrium cross-substitution effect for US exposure to Chinese productivity growth. Consistent with the emphasis in reduced-form empirical studies on impacts in the U.S. market, we find that much of the direct effect of higher Chinese productivity growth occurs within the U.S. (importer's) market. However, we find that a substantial component also occurs within the Chinese (exporter's) market, which highlights the role of both U.S. imports and exports in shaping the impact of the China shock. In comparison, we find smaller third market effects, with the largest third market effects occurring in Singapore, Canada and Japan. This pattern of third market effects is intuitive, as this partial equilibrium cross-substitution effect for the U.S. depends on the product of the share of U.S. income derived from a market (t_{ih}) and the share of that market's expenditure on China (s_{hn}). In line with this intuition, Canada is one of the largest markets for the U.S. (high t_{ih}). Although Singapore and Japan are smaller markets for the U.S. (lower t_{ih}), they have relative high shares of expenditure on China (high s_{nh}), and hence increased Chinese competitiveness has a large impact on US sales within these markets.

While, for ease of interpretation, we illustrate the contributions of the importer's market, exporter's market and third markets using the partial equilibrium cross-substitution effect, the more subtle general equilibrium interactions that occur through the cross-substitution and market-size effects also take place in these three groups of markets, as discussed in Section 3.5 and captured in our overall exposure measures above.

Figure 10: USA Exposure to Partial Equilibrium Cross-Substitution Effect from China, 2010

(a) Importer, Exporter and Third Markets

(b) Individual Third Markets



Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3.

5.4 Sectoral Linkages and Income and Welfare Exposure 1970-2012

We now present evidence from our extensions to incorporate multiple sectors and input-output linkages. In Subsection 5.4.1, we compare the aggregate predictions of the single-sector, multi-sector and input-output specifications, examining the extent to which introducing industry comparative advantage and input-output linkages affects aggregate predictions for income and welfare. In Subsection 5.4.2, we examine the new disaggregated sector-level predictions of these extensions, in which even productivity shocks that are common across sectors in one country can have heterogeneous effects on sectors in its trade partners, depending on the extent to which they compete with one another in sector output markets or source intermediate goods from one another in input markets.

5.4.1 Aggregate Income and Welfare Exposure

In our baseline single-sector Armington model, changes in country welfare in response to foreign productivity shocks occur through changes in the factoral terms of trade. In contrast, in our multi-sector extension from Section 4.4, the terms of trade is also influenced by patterns of comparative advantage across sectors. In particular, the effect of productivity growth in any one foreign country on home welfare depends on the extent to which that foreign country has a similar or dissimilar pattern of comparative advantage across sectors to the home country. In our input-output extension from Section 4.5, this effect of foreign productivity growth on home welfare is further complicated, because domestic comparative advantage across sectors now depends in part on foreign comparative advantage across sectors, through the input-output structure of production.

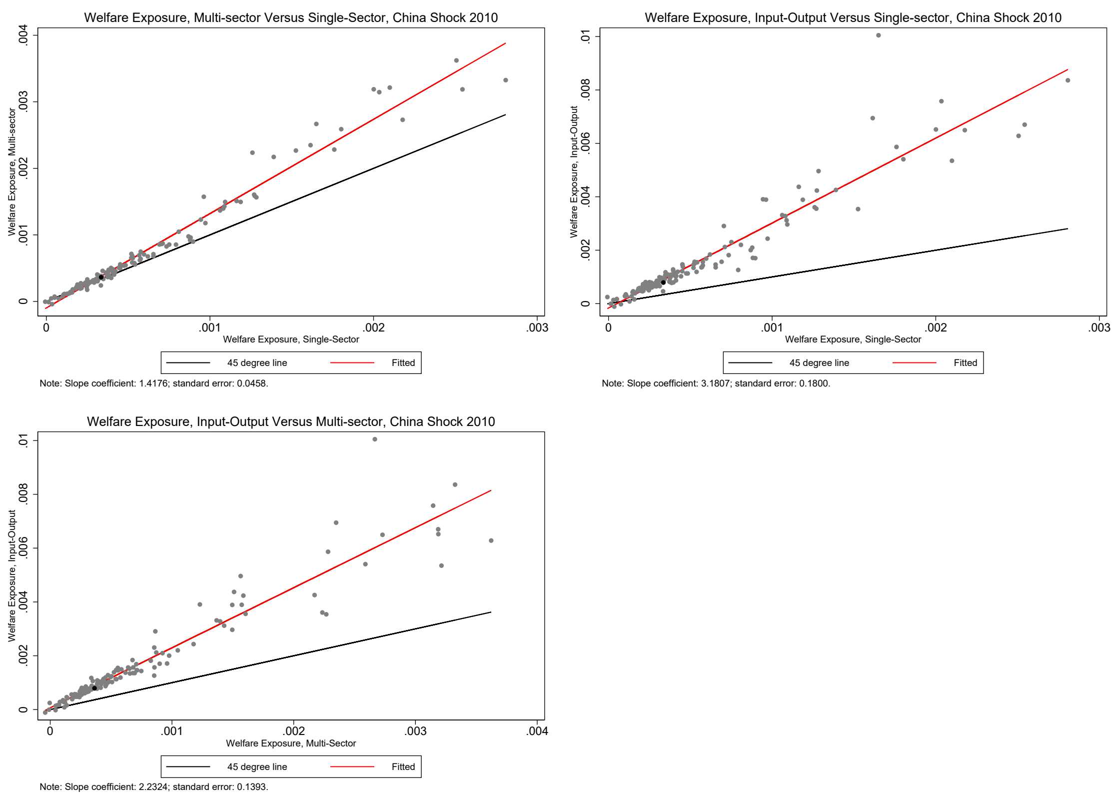
These differences in the determinants of the terms of trade in the three models are reflected in the elements of the trade matrices ($\mathbf{S}$, $\mathbf{T}$ and $\mathbf{M}$) that determine the cross-substitution, market-size and cost-of-living effects. Therefore, although all of these models share the same friends-and-enemies representation of income and welfare exposure, the way in which this representation is constructed differs between them. In the multi-sector model, the elements of the $\mathbf{S}$ and $\mathbf{T}$ matrices in equation (48) now depend on the product of the share of country n's overall expenditure on industry k (α_n^k) times its share of expenditure on country i within that industry (s_{ni}^k). If industries differ in size and vary in importance in the trade between different bilateral pairs of countries as a result of comparative advantage, the resulting elements of these matrices differ from those in the single-sector model. These differences in the elements of the $\mathbf{S}$ and $\mathbf{T}$ matrices in turn induce corresponding differences in the elements of the $\mathbf{M}$ matrix, which depend on the products of $s_{nh}^k t_{hi}^k$ for all markets h. In the input-output model, the elements of all three matrices in equations (53), (54) and (55), respectively, must be further adjusted to take into account the network structure of production, using the observed industry-to-industry flows in the input-output matrix. For the $\mathbf{S}$ and $\mathbf{T}$ matrices that capture the share of an importer's expenditure on each exporter and the share of an exporter's income derived from each importer, respectively, this is largely a matter of accounting. We take into account that the gross value of trade from exporter i to importer n in industry k includes not only the direct value-added created in this exporter and industry but also indirect value-added created in previous stages of production. For the $\mathbf{M}$ matrix, this adjustment also takes into account that the effect of a foreign productivity shock differs depending on whether it reduces intermediate input costs or competitors' output prices.

We now examine the implications of these differences in the $\mathbf{S}$, $\mathbf{T}$ and $\mathbf{M}$ matrices across the three models for their aggregate predictions for countries' welfare exposure to foreign productivity shocks. In our input-output model, we use a common input-output table for all countries from CP, which implies common industry expenditure shares across all countries in traded and non-traded sectors. To ensure a fair comparison across the three models, we make the same assumption of common industry expenditure shares across countries in the multi-sector model, and we incorporate non-traded goods into all three of our models.[12] In Figure 11, we display countries' welfare exposure to a Chinese productivity shock in 2010 (excluding China's own exposure). In the top-left panel, we show the multi-sector model versus the single-sector model; in the top-right panel, we display the input-output model versus the single-sector model; and in the bottom-left panel, we report the input-output model versus the multi-sector model. We find strong correlations between the aggregate predictions of all three models, which are statistically significant

[12] Therefore, our single-sector model in this section features a single traded sector and a single non-traded sector, whereas our multi-industry and input-output models incorporate many disaggregated sectors.

at conventional critical values. In the top-left panel, we find that aggregate welfare responds more strongly to foreign productivity growth in the multi-sector model than the single-sector model (slope coefficient of 1.42), consistent with an additional margin of adjustment in the multi-sector model (industry comparative advantage). In the top-right and bottom-left panels, we find even stronger responses of aggregate welfare to foreign productivity growth in the input-output model than in either of the other models (slope coefficients of 3.18 and 2.23 respectively), consistent with input-output linkages magnifying the effects of productivity improvements.

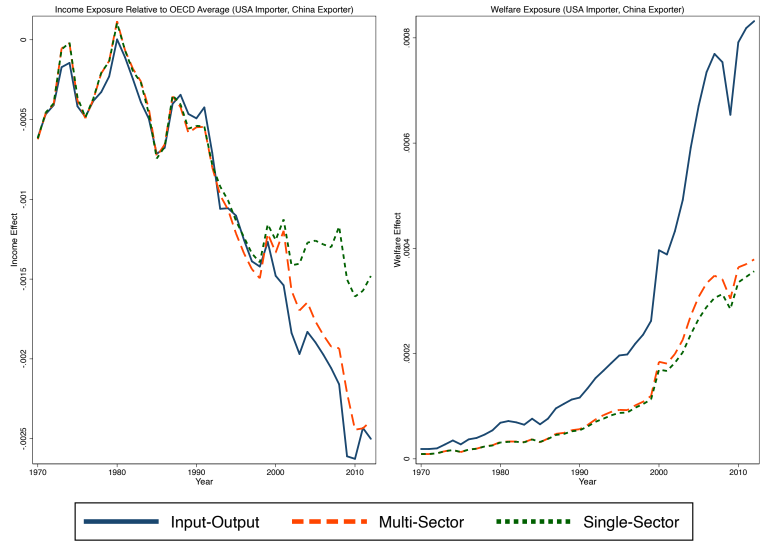
Figure 11: Aggregate Welfare Exposure in the Single-Sector, Multi-Sector and Input-Output Models



Source: NBER World Trade Database and authors' calculations using the single-sector model from Section 3, the multi-sector model from Section 4.4, and the input-output model from Section 4.5.

In Figure 12, we examine the impact of Chinese productivity growth on aggregate U.S. income and welfare over our whole sample period. To ensure that our results for income exposure are invariant to the choice of numeraire, we display the income effect relative to the income-weighted average for all OECD countries. Consistent with our results for all pairs of countries above, we again find similar aggregate welfare predictions across all three models. In each case, we find that Chinese productivity growth has an increasingly negative effect on aggregate US income relative to the OECD average, but an increasingly positive effect on aggregate US welfare, highlighting the powerful cost of living effect in these quantitative trade models. Comparing the single-sector and multi-sector models, we find a substantially more negative effect of Chinese productivity growth on US relative income once we take industry specialization into account. As we move from the multi-sector model to the input-output model, we find a much larger positive effect of Chinese productivity growth on US welfare, again highlighting the potential for input-output linkages to magnify the effects of productivity improvements.

Figure 12: Aggregate Income and Welfare Exposure to Chinese Productivity Growth

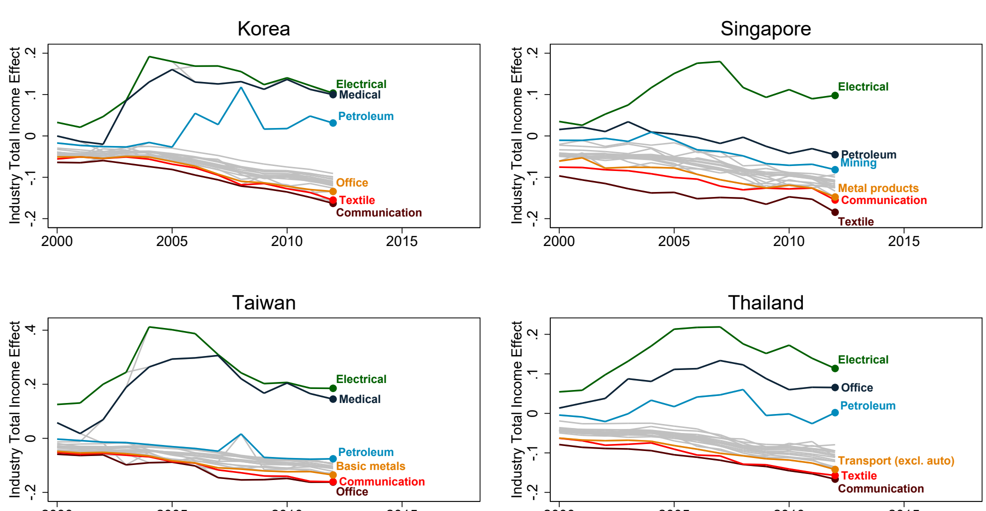


Source: NBER World Trade Database and authors' calculations using the single-sector model from Section 3, the multi-sector model from Section 4.4, and the input-output model from Section 4.5.

5.4.2 Sector Income Exposure

Even though all three models have relatively similar aggregate predictions for income and welfare, the multi-sector and input-output models have additional disaggregated predictions for sector income, as summarized for the multi-sector model in equation (50) in Section 4.4 above. In this section, we briefly illustrate these disaggregated predictions for sector income by considering the impact of Chinese productivity growth on nearby South-East Asian countries and other resource-rich emerging economies, using the input-output model from Section 4.5 above. As discussed for the aggregate income effect above, our choice of world GDP as numeraire implies that a productivity shock that raises a country's own income tends to reduce income in other countries (in order to hold world GDP constant).

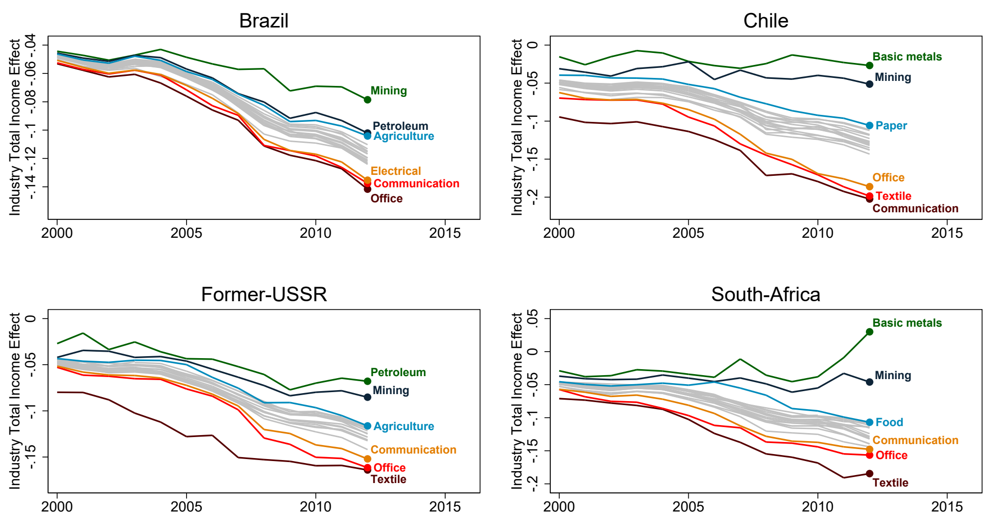
Figure 13: Industry Sales Exposure to Chinese Productivity Growth FOR



Source: NBER World Trade Database and authors' calculations using the input-output model from Section 4.5.

For both the nearby South-East Asian countries (Figure 13) and the resource-rich emerging economies (Figure 14) we find some of the most negative effects for the Textiles sector. In contrast, we find striking differences between the two groups of countries in the sectors with the most positive or least negative income effects. For the nearby South-East Asian countries, the sectors that benefit most from Chinese productivity growth include the Electrical, Medical and Office sectors, consistent with input-output linkages between related sectors through global value chains in Factory Asia. However, for the resource-rich emerging economies, the sectors that benefit most include the Mining, Agricultural and Basic Metals sectors, consistent with a form of "Dutch Disease," in which the growth of resource-intensive sectors propelled by Chinese demand competes away factors of production from less resource-intensive sectors. Taken together, this pattern of results highlights that even common productivity growth across sectors can have subtle and heterogeneous effects on individual sectors in foreign trade partners, depending on patterns of comparative advantage and input sourcing.

Figure 14: USA Industry Sales Exposure to Chinese Productivity Growth



Source: NBER World Trade Database and authors' calculations using the input-output model from Section 4.5.

6 Economic and Political Friends and Enemies

We now use our bilateral measures of exposure to productivity shocks to provide evidence on the political economy debate about the extent to which increased conflict of economic interests between countries necessarily involves heightened political tension between them. In Subsection 6.1, we introduce the data on countries' bilateral political attitudes towards one another. In Subsection 6.2, we examine the relationship between these bilateral political attitudes measures and our economic exposure measures.

6.1 Measuring Bilateral Political Attitudes

Building on a large literature in political science, we measure bilateral political attitudes between countries using two different data sources. First, we use voting behavior in the United Nations General Assembly (UNGA) to reveal the bilateral similarity of countries' foreign policies. Second we use measures of strategic rivalries, as classified by Thompson (2001) and Colaresi et al. (2010), based on contemporary perceptions by political decision makers of whether countries regard one another as competitors, a source of actual or latent threats, or enemies.

6.1.1 United Nations General Assembly Votes

The ultimate source for our UN voting data is Voeten (2013), which reports non-unanimous plenary votes in the United Nations General Assembly (UNGA) from 1962-2012, and includes on average around 128 votes each year. We use the version of these data from the Chance-Corrected Measures of Foreign Policy Similarity (FPSIM) database, as reported in Häge (2017).[13] Countries are recorded as either voting "no" (coded 1), "abstain" (coded 2) or "yes" (coded 3).

We use several measures of bilateral voting similarity constructed from these data, as discussed in further detail in Section G.2.1 of the online appendix. Our first and simplest measure is the S-score of Signorino and Ritter (1999), which equals one minus the sum of the squared actual deviation between a pair of countries' votes scaled by the sum of the squared maximum possible deviations between their votes. By construction, this S-score measure of bilateral voting similarity is bounded between minus one (maximum possible disagreement) and one (maximum possible agreement).

A limitation of this S-score measure is that is does not control for properties of the empirical distribution function of country votes. In particular, country votes may align by chance, such that the frequency with which any two countries agree on a "yes" depends on the frequency with which each country individually votes "yes." Therefore, we also consider two alternative measures of bilateral voting similarity that control in different ways for properties of the empirical distribution of votes. First, the π-score of Scott (1955) adjusts the observed variability of the countries' voting similarity using the variability of each country's own votes around the average vote for the two countries taken together. Second, the κ-score of Cohen (1960) adjusts this observed variability of the countries' voting similarity with the variability of each country's own votes around its own average vote.

Both the π-score and κ-score have an attractive statistical interpretation, as discussed further in Krippendorf (1970), Fay (2005) and Häge (2011). In the case of binary (0,1) voting outcomes, these indices reduce to the form of $1 - (D_o/D_e)$, where D_o is the observed frequency of agreement and D_e is the expected frequency of agreement. The key difference between the two indices is in their assumptions about the expected frequency of agreement. The π-score estimates the expected frequency of agreement using the average of the two countries' marginal distributions of votes. In contrast, the κ-score estimates the expected frequency of agreement using each country's own individual marginal distribution of votes. All three of these measures of foreign policy similarity are necessarily symmetric, whereas our economic measures of exposure to productivity shocks are potentially asymmetric, because country n's exposure to country i is not necessarily the same as i's exposure to n.

Finally, as Bailey et al. (2017) point out, measures based on dyadic similarity of vote choices—such as the S, π, and κ scores—do not account for the heterogeneity in resolutions being voted on. As a result, these measures could incorrectly attribute changes in agenda as changes in state preferences. To resolve this issue, Bailey et al. (2017) apply spatial voting models from the roll call literature to estimate each country's political preferences embedded in its UN votes. The outcome of this statistical procedure is a time-varying, one-dimensional measure called "ideal points", which reflects each country's preference.[14] Bailey et al. (2017) show that ideal points consistently capture the position of states vis-à-vis a US-led liberal order. We derive a measure of bilateral distance by taking the absolute difference

[13] See https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/ALVXLM

[14] Specifically, Bailey et al. (2017)'s methodology identifies preference change over time by exploiting duplicate resolutions that are voted repeatedly in consecutive sessions. This methodology also weights resolutions based on how much they reflect the main policy preference dimension in order to ensure that ideal points are not heavily influenced by resolutions that reflect idiosyncratic factors.

between the ideal points of countries i and j in year t.

6.1.2 Strategic Rivalry

Our second set of measures of countries' bilateral attitudes are indicator variables that pick up whether country i is a strategic rival of j in year t, as classified by Thompson (2001) and Colaresi et al. (2010). These rivalry measures capture the risk of conflict with a country of significant relative size and military strength, based on contemporary perceptions by political decision makers, gathered from historical sources on foreign policy and diplomacy. Specifically, rivalries are identified by whether two countries regard each other as competitors, a source of actual or latent threats that pose some possibility of becoming militarized, or enemies.[15]

Prior research has shown that rivalry occurs much more frequently than actual wars (Colaresi et al. 2010, Aghion et al. 2018). In our sample from 1970-2012, a total of 42 countries have had at least one strategic rival; 74 country-pairs have been strategic rivals at some point; and the total number of country-pair-years that exhibit strategic rivalry is 2,452. China, for instance, is classified as being in strategic rivalry with the U.S. (1970–1972 and 1996–present), India (the entire sample period), Japan (1996–present), the former Soviet Union (1970–1989), and Vietnam (1973–1991).

6.2 Bilateral Political Attitudes and Economic Exposure

We now examine the relationship between these measures of bilateral political attitudes and our friends-and-enemies sufficient statistics. We estimate the following regression specification for importer n, exporter i, and time t:

$$\mathbf{A}_{nit} = \beta \mathbf{U}_{nit} + \eta_{ni} + d_t + \epsilon_{nit}, \tag{59}$$

where $\mathbf{A}_{nit}$ is one of our measures of bilateral foreign policy similarity; $\mathbf{U}_{nit}$ is our friends-and-enemies welfare exposure measure; η_{ni} is an exporter-importer fixed effect that controls for time-invariant unobserved heterogeneity that is specific to an exporter-importer pair and affects both political attitudes and economic exposure; d_t are year dummies; in some specifications, we replace the year fixed effects with exporter-year fixed effects and importer-year fixed effects; the exporter-year fixed effects control for the observation for which we shock productivity and capture unobserved shocks that affect an exporter's welfare exposure and political attitudes across all importers in a given year; the importer-year fixed effects control for unobserved shocks that affect an importer's welfare exposure and political attitudes across all exporters in a given year; ϵ_{nit} is a stochastic error; and we cluster the standard errors in all specifications by exporter-importer pair to allow for serial correlation in this error term over time.

Our inclusion of exporter-importer and year fixed effects implies that the regression specification (59) has a difference-in-difference interpretation, where the first difference is over time and the second difference is between exporter-importer pairs. The key coefficient of interest β is identified from differential changes within exporter-importer pairs over time: we examine whether as an exporter-importer pair becomes more economically friendly in terms of the welfare effects of productivity growth, it also becomes more politically friendly in terms of foreign policy similarity. A concern about estimating equation (59) using OLS is the potential for reverse causality: unobserved changes over time in countries' bilateral attitudes towards one another in the error term (ϵ_{nit}) could lead to changes in bilateral trade between them. These changes in bilateral trade could in turn affect bilateral welfare exposure ($\mathbf{U}_{nit}$),

[15] Colaresi et al. (2010) further refine the data to distinguish between three types of rivalries: spatial, where rivals contest the exclusive control of a territory; positional, where rivals contest relative shares of influence over activities and prestige within a system or subsystem; and ideological, where rivals contest the relative virtues of different belief systems relating to political, economic or religious activities.

thereby inducing a correlation between welfare exposure and the error term. To address this concern, we use a specification that follows Frankel and Romer (1999) in using predicted trade flows abstracting from this variation as an instrument. In particular, we estimate the following gravity equation for bilateral trade (x_{nit}) between countries for each year separately:

$$x_{nit} = \chi_{nt} \xi_{it} \text{dist}_{ni}^{\phi_t} \varpi_{nit}, \tag{60}$$

where χ_{nt} is an importer-year fixed effect; ξ_{it} is an exporter-year fixed effect; ϕ_t is the time-varying coefficient on distance; and ϖ_{nit} is a stochastic error. We estimate this gravity equation using the Poisson Pseudo Maximum Likelihood estimator of Santos Silva and Tenreyro (2006), which yields theory-consistent estimates of the fixed effects, as shown in Fally (2015). To abstract from changes over time in countries' bilateral attitudes for one another, we use the fitted values from this regression ($\hat{x}_{nit} = \hat{\chi}_{nt} \hat{\xi}_{it} \text{dist}_{ni}^{\hat{\phi}_t}$) to construct predicted expenditure shares ($\hat{s}_{nit} = \hat{x}_{nit} / \sum_{m=1}^{N} \hat{x}_{mit}$), thereby removing the bilateral error term ϖ_{nit}. In our class of models, these expenditure shares (s_{nit}) determine income shares (t_{int}), cross-substitution (m_{int}), and hence welfare exposure ($\mathbf{U}_{nit}$). Therefore, we use the predicted expenditure shares ($\hat{s}_{nit}$) to instrument welfare exposure ($\mathbf{U}_{nit}$) in equation (59). Even after conditioning on importer-year and exporter-year fixed effects, there is bilateral variation over time in these predicted expenditure shares ($\hat{s}_{nit}$), because of the time-varying coefficient on distance (ϕ_t).

In Table 1, we report results of estimating equation (59) using both sets of attitudes measures (UN voting and strategic rivalries) and our welfare exposure measure. We estimate this relationship using two-stage least squares, instrumenting welfare exposure using our predicted trade shares. Columns (1)-(2) use the S-score; Columns (3)-(4) use the π-score; Columns (5)-(6) use the κ-score; Columns (7)-(8) use the distance in ideal points; and Columns (9)-(10) use strategic rivalries (all types). In each of these pairs of specifications, the first column ((1), (3), (5), (7) and (9)) includes only the exporter-importer and year fixed effects; the second column ((2), (4), (6), (8) and (10)) augments this specification with exporter-year and importer-year fixed effects. Panel A uses welfare exposure from the single-sector model from Section 3 above; Panel B uses welfare exposure from the multi-sector model from Section 4.4 above; and Panel C uses welfare exposure from the input-output model from Section 4.5 above.

Across Columns (1)-(6), we find positive and statistically significant coefficients in all specifications, implying that as countries become greater economic friends in terms of the welfare effects of their productivity growth, they also become greater political friends in terms of their voting similarity in the UNGA. Consistent with these results, in Columns (7)-(10), we find negative and statistically significant coefficients in all specifications, implying that as countries become greater economic friends in terms of the welfare effects of their productivity growth, they again become greater political friends in terms of having smaller bilateral distances from the US-led liberal order and a lower propensity to be strategic rivals. Beneath the coefficient and standard error for each specification, we report the first-stage F-statistic. These first-stage F-statistics take the same value across Columns (1), (3) and (5) and Columns (2), (4) and (6), because the first-stage regression specification (welfare exposure on the instrument) and sample size is the same across the different political attitudes measures used in the second-stage regression. Although these first-stage F-statistics naturally fall in the specifications including importer-year and exporter-year fixed effects, they remain well above the conventional threshold of 10.[16]

[16] In Section G.2.2 of the online appendix, we show that we find a similar pattern of results for strategic rivalries when we consider the different types of strategic rivalry separately (spatial, positional and ideological). Therefore, this relationship between the similarity of economic and political interests holds regardless of which of these different dimensions of strategic rivalries we consider.

Table 1: Bilateral Political Attitudes (Voting Similarity and Strategic Rivalry) and Welfare Exposure

Political Outcome	Voting Similarity-S		Voting Similarity-κ		Voting Similarity-π		Distance in ideal points		Strategic rivalry (any type)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Panel A: Welfare exposure in single-sector model										
$\mathbf{U}^{Single-sector}$	9.736***	11.78***	23.09***	21.54***	26.09***	24.79***	-37.26***	-32.97***	-4.741***	-5.379***
	(2.738)	(2.446)	(4.644)	(4.434)	(5.403)	(4.967)	(10.82)	(8.535)	(1.782)	(1.960)
First-stage F	98.08	78.50	98.08	78.50	98.08	78.50	110.4	86.86	99.50	78.52
Panel B: Welfare exposure in multi-sector model										
$\mathbf{U}^{Multi-sector}$	9.635***	11.66***	22.85***	21.32***	25.82***	24.54***	-36.87***	-32.64***	-4.695***	-5.331***
	(2.710)	(2.428)	(4.610)	(4.407)	(5.356)	(4.933)	(10.73)	(8.487)	(1.766)	(1.944)
First-stage F	98.61	78.76	98.61	78.76	98.61	78.76	110.4	86.67	99.36	78.20
Panel C: Welfare exposure in input-output model										
$\mathbf{U}^{Input-Output}$	20.41***	26.14***	48.42***	47.80***	54.69***	55.00***	-76.94***	-72.19***	-9.831***	-11.83***
	(5.211)	(4.638)	(7.965)	(8.192)	(9.379)	(9.156)	(20.24)	(16.74)	(3.495)	(4.054)
First-stage F	161.1	113.6	161.1	113.6	161.1	113.6	163.5	114.3	173.6	119.8
Specification: 2SLS										
Exp × Imp	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
Exp × Year	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Imp × Year	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
No. of Obs.	585884	585884	585884	585884	585884	585884	567790	567790	610954	610954
No. of Clusters	14721	14721	14721	14721	14721	14721	14479	14479	14761	14761

Standard errors in parentheses are clustered at country-pair level
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Taken together, these empirical results are consistent with the view that increased conflict of economic interests between countries leads to heightened political tension between them. As these measures of bilateral political attitudes are measured entirely independently of any of these quantitative trade models, they also provide an external validation that our friends-and-enemies measures of welfare exposure are systematically related to independent measures of countries' attitudes towards one another.

7 Conclusions

The closing decades of the twentieth century saw large-scale changes in the relative economic size of nations, with China's rapid economic growth transforming it into a major trading nation. A classic question in international economics is the implications of such economic growth for the income and welfare of trade partners. A related question in political economy is the extent to which these large-scale changes in relative economic size necessarily involve political tension and realignments. We provide new theory and evidence on both of these questions by developing bilateral "friends" and "enemies" measures of countries' income and welfare exposure to foreign productivity shocks that can be computed using only observed trade data. We show that these measures are exact for small productivity shocks in the leading class of international trade models characterized by a constant trade elasticity. For large productivity shocks, we characterize the quality of the approximation in terms of the properties of the observed trade data, and show that for the magnitude of the productivity shocks implied by the observed data, our exposure measures are almost visibly indistinguishable from the predictions of the full non-linear solution of the model. Our approach admits a large number of extensions and generalizations, including multiple sectors, input-output linkages and economic geography (factor mobility). Using our methods, we derive bounds for the sensitivity of countries' exposure to foreign productivity shocks to departures from a constant trade elasticity.

We contribute to the recent revolution in international trade of the development of quantitative trade models. A key advantage of these quantitative models is that they are rich enough to capture first-order features of the data, such as a gravity equation for bilateral trade, and yet remain sufficiently tractable as to be amenable to counterfactual analysis, with a small number of structural parameters. A key challenge is that these models are highly non-linear, which can make it difficult to understand the economic explanations for quantitative findings for particular countries or industries. A key contribution of our bilateral friends-and-enemies measures is to allow researchers to connect quantitative results to the key underlying economic mechanisms in the model: the cross-substitution effect, where an increase in the competitiveness of a foreign country leads consumers in all markets to substitute away from all other nations; a market-size effect, where an increase in income in foreign markets raises demand for all nations' goods; and a cost-of-living effect, where an increase in the competitiveness of a country's goods reduces the cost of living in all countries. As our linearization uses standard matrix inversion techniques, we find that it is around 70,000 faster than solving the full non-linear solution of the model. Therefore, our methods are well suited to applications where large numbers of counterfactuals are required, and facilitate comparisons of these counterfactuals across alternative quantitative frameworks, such as our single-sector, multi-sector and input-output models.

We use our friends-and-enemies exposure measures to examine the global incidence of productivity growth in each country on income and welfare for more than 140 countries over more than 40 years from 1970-2012 (around one million bilateral comparative statics). We find a substantial and statistically significant increase in both the mean and dispersion of welfare exposure to foreign productivity growth over our sample period, consistent with increasing globalization enhancing countries' economic dependence on one another. We also observe large-scale changes in bilateral networks of welfare exposure between nations. We find that the cross-substitution, market-size and cost-of-living effects are all quantitatively important for the welfare impact of foreign productivity growth, and the general equilibrium forces in this class of quantitative trade models are large relative to the partial equilibrium effects of productivity growth. Whether we consider the single-sector, multi-sector or input-output models, we find that Chinese productivity growth has reduced aggregate U.S. income, both relative to world GDP and other OECD countries. Nevertheless, we find that Chinese productivity growth has raised aggregate U.S. welfare, highlighting the strength of the cost-of-living effect in these quantitative trade models. Consistent with the idea that conflicting economic interests can spawn political discord, we find that as countries become greater economic friends in terms of the welfare effects of their productivity growth, they become greater political friends in terms of the similarity of their foreign policy stances, as measured by United Nations voting patterns and strategic rivalries.

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Online Appendix for "International Friends and Enemies" (Not for Publication)

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Table of Contents

A Introduction

In this section of the online appendix, we report the detailed derivations for the results reported in the paper and further supplementary results. In Section B, we report the proofs of the propositions in the paper. In Section C, we consider the Armington model with a general homothetic utility function in which goods are differentiated by country of origin from Section 2 of the paper. In Section D, we examine the special case of this model that falls within the class of models considered by Arkolakis, Costinot and Rodriguez-Clare (2012, henceforth ACR), which satisfy the four primitive assumptions of (i) Dixit-Stiglitz preferences; (ii) one factor of production; (iii) linear cost functions;

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and (iv) perfect or monopolistic competition; as well as the three macro restrictions of (i) a constant elasticity import demand system, (ii) a constant share of profits in income, and (iii) balanced trade, as discussed in Section 3 of the paper. Although for convenience of exposition we focus in the paper and Section D of this online appendix on the single-sector Armington model, in Section E we show the same income and welfare exposure measures apply for all models in the ACR class with a constant trade elasticity. In Subsection E.1, we derive our exposure measures in a version of the Eaton and Kortum (2002) model. In Subsection E.2, we derive these exposure measures in the Krugman (1980) model.

In Section F, we consider a number of extensions of our baseline friend-enemies results with a constant trade elasticity, as in Section 4 of the paper. In Subsection F.1, we derive the corresponding friend-enemy representation allowing for both productivity and trade cost shocks. In Section F.2, we relax one of the ACR macro restrictions to allow for trade imbalance. In Section F.3, we relax another of the ACR macro restrictions to consider small deviations from a constant elasticity import demand system. In Section F.4, we show that our results generalize to a multi-sector version of the constant elasticity Armington model. In Section F.5, we show that they also hold in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). In Section F.6, we further extend the multi-sector Armington model from Section F.4 to introduce input-output linkages following Caliendo and Parro (2015). Finally, in Section F.7, we show that our results also hold for economic geography models with factor mobility, including Helpman (1998), Redding and Sturm (2008), Allen and Arkolakis (2014), Ramondo et al. (2016) and Redding (2016).

In Section G, we provide further details on the empirical specifications reported in the paper, as well as additional supplementary empirical results. Section H contains the data appendix.

B Proofs of Propositions

B.1 Proof of Lemmas 1 and 3

Lemma 1 imposes trade balance, with market clearing conditions $w_i\ell_i = \sum_n s_{ni}w_n\ell_n$. Define $q_i \equiv \frac{w_i\ell_i}{\sum_n w_n\ell_n}$; we can rewrite market clearing condition as $q_i = \sum_n s_{ni}q_n$, and, in matrix form, $\mathbf{q}' = \mathbf{q}'\mathbf{S}$, proving that $\mathbf{q}$ is a left-eigenvector of $\mathbf{S}$ with eigenvalue 1. That $\mathbf{q}'$ is the unique positive left-eigenvector of $\mathbf{S}$ follows from Perron-Frobenius theorem. Under free-trade, every row of $\mathbf{S}$ is identical, and $\sum_n q_n s_{ni} = s_{1i}$ for all i. The vector $[s_{11}, s_{12}, \cdots, s_{1N}]$ is therefore also left-eigenvector of $\mathbf{S}$ with eigenvalue 1, and since its entries are all positive, it must be equal to $\mathbf{q}$. Likewise, market-clearing can also be written as $w_n\ell_n = \sum_i t_{in}w_i\ell_i$, which is equivalent to, in matrix form, $\mathbf{q}' = \mathbf{q}'\mathbf{T}$. The remaining claims about $\mathbf{T}$ follow analogously.

Lemma 3 introduces trade imbalances to the market clearing conditions, with $w_i\ell_i = \sum_n \left[s_{ni}w_n\ell_n + \bar{d}_n\right]$. Let $q_i \equiv w_i\ell_i / \left(\sum_n w_n\ell_n\right)$ and $e_i \equiv \left(w_i\ell_i + \bar{d}_n\right) / \left(\sum_n w_n\ell_n\right)$. Dividing the market clearing condition by $\left(\sum_j w_j\ell_j\right)$, we have

$$\frac{w_i\ell_i}{\sum_j w_j\ell_j} = \sum_n \left[s_{ni}\frac{w_n\ell_n + \bar{d}_n}{\sum_j w_j\ell_j}\right] \iff q_i = \sum_n s_{ni}e_n \iff \mathbf{q}' = \mathbf{e}'\mathbf{S}.$$

Let $d_i \equiv q_i/e_i$ and $\mathbf{D} \equiv Diag\left(\mathbf{d}\right)$. Note $\mathbf{q}' = \mathbf{e}'\mathbf{D}$ and $\mathbf{q}'\mathbf{D}^{-1} = \mathbf{e}'$; thus the market clearing condition can be re-written as

$$\mathbf{e}'\mathbf{D} = \mathbf{e}'\mathbf{S} \iff \mathbf{e}' = \mathbf{e}'\mathbf{S}\mathbf{D}^{-1}$$

and

$$\mathbf{q}' = \mathbf{e}'\mathbf{S} \iff \mathbf{q}' = \mathbf{q}'\mathbf{D}^{-1}\mathbf{S}.$$

$\mathbf{q}$ is therefore the unique positive left-eigenvector of $\mathbf{D}^{-1}\mathbf{S}$ with eigenvalue 1, and $\mathbf{e}'$ is the unique positive left-eigenvector of $\mathbf{S}\mathbf{D}^{-1}$ with eigenvalue one. The remaining claims about $\mathbf{T}$ follow analogously.

B.2 Proof of Lemma 2

Note that $\mathbf{Z} \equiv \frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}$ is a row-stochastic matrix and represents a Markov chain; its eigenvector $\mathbf{q}$ represents the stationary distribution of the Markov chain. Invertibility of $(\mathbf{I}-\mathbf{V})$ follows from convergence of the power series $\sum_{k=0}^{\infty}\mathbf{V}^k \equiv \sum_{k=0}^{\infty}(\mathbf{Z}-\mathbf{Q})^k$, which we show now. By construction, $\mathbf{QZ} = \mathbf{Q}$ and $\mathbf{ZQ} = \mathbf{Q}$. Using these two relations, we can show by induction that $(\mathbf{Z}-\mathbf{Q})^k = \mathbf{Z}^k - \mathbf{Q}$ for any integer $k > 0$. That $\|\mathbf{Z}^k - \mathbf{Q}\| \le c \cdot |\mu|^k$, where μ is the largest eigenvalue of $\mathbf{V}$ in terms of absolute value (and the second-largest eigenvalue of $\mathbf{Z}$), follows from standard results on Markov chains (e.g., see Rosenthal (1995)).

B.3 Proof of Proposition 3

We repeatedly apply the following approximations:

$$\ln(1+x) \approx x - \frac{x^2}{2}, \quad x - 1 \approx \ln x + \frac{(\ln x)^2}{2}$$

$$\begin{aligned}
\ln\left(\sum_i p_i x_i\right) &\approx \sum_i p_i (x_i - 1) - \frac{\left(\sum_i p_i (x_i - 1)\right)^2}{2} \\
&\approx \sum_i p_i \left(\ln x_i + \frac{(\ln x_i)^2}{2}\right) - \frac{\left(\sum_i p_i \ln x\right)^2}{2} \\
&= \mathbb{E}_p[\ln x_i] + \frac{\mathbb{V}_p(\ln x_i)}{2}
\end{aligned}$$

Let $\tilde{x} \equiv \ln \hat{x}$. The hat-algebra with only TFP shocks can be written as

$$\hat{w}_i = \sum_n T_{in}\hat{w}_n \frac{\hat{c}_i^{-\theta}}{\sum_k s_{nk}\hat{c}_k^{-\theta}}, \quad \text{where } \hat{c}_i \equiv \widehat{w_i/z_i}$$

Taking logs,

$$\begin{aligned}
\tilde{w}_i &= \ln\left(\sum_n T_{in}\hat{w}_n \frac{\hat{c}_i^{-\theta}}{\sum_k s_{nk}\hat{c}_k^{-\theta}}\right) \\
&= \mathbb{E}_{T_i}\left[\ln\left(\hat{w}_n \frac{\hat{c}_i^{-\theta}}{\sum_k s_{nk}\hat{c}_k^{-\theta}}\right)\right] + \frac{\mathbb{V}_{T_i}\left(\ln\left(\hat{w}_n \frac{\hat{c}_i^{-\theta}}{\sum_k s_{nk}\hat{c}_k^{-\theta}}\right)\right)}{2} \\
&= \mathbb{E}_{T_i}\left[\tilde{w}_n - \theta\tilde{c}_i + \mathbb{E}_{S_n}[\theta\tilde{c}_k] - \frac{\mathbb{V}_{S_n}[\theta\tilde{c}_k]}{2}\right] + \frac{\mathbb{V}_{T_i}\left(\tilde{w}_n + \mathbb{E}_{S_n}[\theta\tilde{c}_k]\right)}{2} \\
&= -\theta\tilde{c}_i + \mathbb{E}_{T_i}[\tilde{w}_n] + \mathbb{E}_{M_i}[\theta\tilde{c}_k] - \frac{\mathbb{E}_{T_i}\mathbb{V}_{S_n}[\theta\tilde{c}_k]}{2} + \frac{\mathbb{V}_{T_i}\left(\tilde{w}_n + \mathbb{E}_{S_n}[\theta\tilde{c}_k]\right)}{2}
\end{aligned}$$

To re-write the second-order terms explicitly as a function of the productivity shocks—thereby deriving the Hessian—we express $\theta\tilde{c}_k$ and $\tilde{w}_n + \mathbb{E}_{S_n}[\theta\tilde{c}_k]$ in terms of productivity shocks to first-order. To do so, note that the first-order approximation is

$$\tilde{\mathbf{w}} = \mathbf{T}\tilde{\mathbf{w}} + \theta(\mathbf{TS} - \mathbf{I})(\tilde{\mathbf{w}} - \tilde{\mathbf{z}})$$

$$\implies \tilde{\mathbf{w}} = -\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right)\tilde{\mathbf{z}}$$

where $\mathbf{V} \equiv \frac{\mathbf{T}+\theta\mathbf{TS}}{1+\theta} - \mathbf{Q}$. We can therefore rewrite

$$\begin{aligned}
\theta\left(\tilde{\mathbf{w}}-\tilde{\mathbf{z}}\right) &= -\theta\left(\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right)+\mathbf{I}\right)\tilde{z} \\
&= -\theta\left(\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right)+\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{V}\right)\right)\tilde{\mathbf{z}} \\
&= -\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right)\tilde{\mathbf{z}}
\end{aligned}$$

Where we have used the fact $\mathbf{q}'\tilde{\mathbf{z}} = 0$, which follows from CRTS and our normalization that world GDP is constant, to drop $\mathbf{Q}\tilde{\mathbf{z}}$ from the RHS.

We further have

$$\begin{aligned}
\tilde{\mathbf{w}}+\theta\mathbf{S}\left(\tilde{\mathbf{w}}-\tilde{\mathbf{z}}\right) &= -\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right)\tilde{\mathbf{z}}-\mathbf{S}\left(\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}+\mathbf{Q}\right)\right)\tilde{\mathbf{z}} \\
&= -\frac{\theta}{\theta+1}\left\{\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right)+\mathbf{S}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}+\mathbf{Q}\right)\right\}\tilde{\mathbf{z}}.
\end{aligned}$$

Further, to reduce notational clutter, let $\mathbf{A} \equiv \frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right)$ and $\mathbf{B} \equiv \frac{\theta}{\theta+1}\left\{\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right)+\mathbf{S}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right)\right\}$, $\mathbf{y} \equiv \theta\left(\tilde{\mathbf{w}}-\tilde{\mathbf{z}}\right)$, $\mathbf{x} \equiv \tilde{\mathbf{w}}-\theta\mathbf{S}\left(\tilde{\mathbf{w}}-\tilde{\mathbf{z}}\right)$, thus

$$\mathbf{y} = -\mathbf{A}\tilde{\mathbf{z}}, \quad \mathbf{x} = +\mathbf{B}\tilde{\mathbf{z}}$$

We can now re-write the second-order terms as (let $\mathbf{D}\left(\mathbf{x}\right) \equiv diag\left(\mathbf{x}\right)$ denote the diagonalization operator)

$$\begin{aligned}
& \mathbb{E}_{T_i}V_{S_n}\left[\theta\tilde{c}_k\right]-\mathbb{V}_{T_i}\left(\tilde{w}_n+\mathbb{E}_{S_n}\left[\theta\tilde{c}_k\right]\right) \\
= & \mathbb{E}_{T_i}V_{S_n}\left[y_k\right]-\mathbb{V}_{T_i}\left(x_n\right) \\
= & \sum_n T_{in}\left[\sum S_{nk}y_k^2-\left(\sum S_{nk}y_k\right)^2\right]-\left(\sum_n T_{in}x_n^2-\left(\sum_n T_{in}x_n\right)^2\right) \\
= & \mathbf{y}'\mathbf{D}\left(\left[\mathbf{M}+\mathbf{I}\right]_i\right)\mathbf{y}-\mathbf{y}'\mathbf{S}'\mathbf{D}\left(\mathbf{T}_i\right)\mathbf{S}\mathbf{y}-\left(\mathbf{x}'\mathbf{D}\left(\mathbf{T}_i\right)\mathbf{x}-\mathbf{x}'\mathbf{T}'\mathbf{T}\mathbf{x}\right) \\
= & \tilde{\mathbf{z}}'\mathbf{A}'\mathbf{D}\left(\left[\mathbf{M}+\mathbf{I}\right]_i\right)\mathbf{A}\tilde{\mathbf{z}}-\mathbf{z}'\mathbf{A}'\mathbf{S}'\mathbf{D}\left(\mathbf{T}_i\right)\mathbf{S}\mathbf{A}\tilde{\mathbf{z}}-\left(\tilde{\mathbf{z}}'\mathbf{B}'\mathbf{D}\left(\mathbf{T}_i\right)\mathbf{B}\tilde{\mathbf{z}}-\tilde{\mathbf{z}}'\mathbf{B}'\mathbf{T}_i'\mathbf{T}_i\mathbf{B}\tilde{\mathbf{z}}\right) \\
= & \tilde{\mathbf{z}}'\left(\mathbf{A}'\left(\mathbf{D}\left(\left[\mathbf{M}+\mathbf{I}\right]_i\right)-\mathbf{S}'\mathbf{D}\left(\mathbf{T}_i\right)\mathbf{S}\right)\mathbf{A}-\mathbf{B}'\left(\mathbf{D}\left(\mathbf{T}_i\right)-\mathbf{T}_i'\mathbf{T}_i\right)\mathbf{B}\right)\tilde{\mathbf{z}}.
\end{aligned}$$

Hence we have $\epsilon_i \equiv \tilde{\mathbf{z}}'\mathbf{H}_{f_i}\tilde{\mathbf{z}}$, where

$$\mathbf{H}_{f_i} = -\frac{1}{2}\left(\mathbf{A}^{\mathbf{T}}\left(diag\left(\left[\mathbf{M}+\mathbf{I}\right]_i\right)-\mathbf{S}'diag\left(\mathbf{T}_i\right)\mathbf{S}\right)\mathbf{A}-\mathbf{B}^{\mathbf{T}}\left(diag\left(\mathbf{T}_i\right)-\mathbf{T}_i'\mathbf{T}_i\right)\mathbf{B}\right).$$

B.4 Proof of Proposition 4

Lemma 4. $\left(\mathbf{I}-\mathbf{S}\right)\mathbf{A} = -\left(\mathbf{I}-\mathbf{T}\right)\mathbf{B}$, *where* $\mathbf{A} \equiv \frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right)$ *and* $\mathbf{B} \equiv \frac{\theta}{\theta+1}\left\{\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right)+\mathbf{S}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right)\right\}$.

Proof. By the definition of $\mathbf{A}$ and $\mathbf{B}$:

$$\left(\mathbf{I}-\mathbf{S}\right)\mathbf{A} = \frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{S}\right)\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right)$$

$$\left(\mathbf{I}-\mathbf{T}\right)\mathbf{B} = \frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{T}\right)\left\{\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right)+\mathbf{S}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right)\right\}$$

We have

$$
\begin{aligned}
\frac{\theta+1}{\theta}\left(\left(\mathbf{I}-\mathbf{S}\right)\mathbf{A}+\left(\mathbf{I}-\mathbf{T}\right)\mathbf{B}\right) &= \left(\mathbf{I}-\mathbf{S}\right)\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right) \\
&\quad +\left(\mathbf{I}-\mathbf{T}\right)\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right)+\left(\mathbf{I}-\mathbf{T}\right)\mathbf{S}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right) \\
&= \left(\mathbf{I}-\mathbf{S}+\left(\mathbf{I}-\mathbf{T}\right)\mathbf{S}\right)\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right)+\left(\mathbf{I}-\mathbf{T}\right)\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right) \\
&= \left(\mathbf{I}-\mathbf{TS}\right)\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right)+\left(\mathbf{I}-\mathbf{T}\right)\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right) \\
&= \left(\mathbf{I}-\mathbf{TS}\right)\left(\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right)+\mathbf{I}\right) \\
&\quad +\left(\frac{\theta}{\theta+1}\left(\mathbf{TS}-\mathbf{I}\right)\left(\mathbf{I}-\mathbf{V}\right)^{-1}+\mathbf{I}\right)\left(\mathbf{TS}-\mathbf{I}\right) \\
&= \left(\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{TS}\right)\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right)+\left(\mathbf{I}-\mathbf{TS}\right)\right) \\
&\quad +\left(\frac{\theta}{\theta+1}\left(\mathbf{TS}-\mathbf{I}\right)\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{TS}-\mathbf{I}\right)+\left(\mathbf{TS}-\mathbf{I}\right)\right) \\
&= 0.
\end{aligned}
$$

□

Proof of Proposition 4 To show that the second-order terms average to zero across countries, note

$$
\begin{aligned}
\boldsymbol{q}'\epsilon &= \tilde{\mathbf{z}}^{'}\left(\sum q_i H_{f_i}\right)\tilde{\mathbf{z}} \\
&= -\frac{1}{2}\tilde{\mathbf{z}}^{'}\left(\mathbf{A}^{'}\left(\mathbf{d}-\mathbf{S}'\mathbf{dS}\right)\mathbf{A}-\mathbf{B}^{'}\left(\mathbf{d}-\mathbf{T}'\mathbf{dT}\right)\mathbf{B}\right)\tilde{\mathbf{z}},
\end{aligned}
$$

where $\mathbf{d}\equiv diag\left(\boldsymbol{q}\right)$. We next show $\left(\mathbf{A}^{'}\left(\mathbf{d}-\mathbf{S}'\mathbf{dS}\right)\mathbf{A}-\mathbf{B}^{'}\left(\mathbf{d}-\mathbf{T}'\mathbf{dT}\right)\mathbf{B}\right)$ is a zero matrix. Using Lemma 1, we have

$$
\begin{aligned}
& 2\left(\mathbf{A}^{'}\left(\mathbf{d}-\mathbf{S}'\mathbf{dS}\right)\mathbf{A}-\mathbf{B}^{'}\left(\mathbf{d}-\mathbf{T}'\mathbf{dT}\right)\mathbf{B}\right) \\
= & \mathbf{A}^{'}\left(\mathbf{I}-\mathbf{S}'\right)\mathbf{d}\left(\mathbf{I}+\mathbf{S}\right)\mathbf{A}-\mathbf{B}'\left(\mathbf{I}+\mathbf{T}'\right)\mathbf{d}\left(\mathbf{I}-\mathbf{T}\right)\mathbf{B} \\
& +\mathbf{A}'\left(\mathbf{I}+\mathbf{S}'\right)\mathbf{d}\left(\mathbf{I}-\mathbf{S}\right)\mathbf{A}-\mathbf{B}^{'}\left(\mathbf{I}-\mathbf{T}'\right)\mathbf{d}\left(\mathbf{I}+\mathbf{T}\right)\mathbf{B} \\
& \text{(the next line follows from Lemma 1)} \\
= & -\mathbf{B}^{'}\left(\mathbf{I}-\mathbf{T}'\right)\mathbf{d}\left(\mathbf{I}+\mathbf{S}\right)\mathbf{A}+\mathbf{B}'\left(\mathbf{I}+\mathbf{T}'\right)\mathbf{d}\left(\mathbf{I}-\mathbf{S}\right)\mathbf{A} \\
& -\mathbf{A}'\left(\mathbf{I}+\mathbf{S}'\right)\mathbf{d}\left(\mathbf{I}-\mathbf{T}\right)\mathbf{B}+\mathbf{A}^{'}\left(\mathbf{I}-\mathbf{S}'\right)\mathbf{d}\left(\mathbf{I}+\mathbf{T}\right)\mathbf{B} \\
= & -\left(\mathbf{B}'-\mathbf{B}'\mathbf{T}'\right)\mathbf{d}\left(\mathbf{A}+\mathbf{SA}\right)+\left(\mathbf{B}'+\mathbf{B}'\mathbf{T}'\right)\mathbf{d}\left(\mathbf{A}-\mathbf{SA}\right) \\
& -\left(\mathbf{A}'+\mathbf{A}'\mathbf{S}'\right)\mathbf{d}\left(\mathbf{B}-\mathbf{TB}\right)+\left(\mathbf{A}^{'}-\mathbf{A}^{'}\mathbf{S}'\right)\mathbf{d}\left(\mathbf{B}+\mathbf{BT}\right) \\
= & 2\left(\mathbf{B}'\left(\mathbf{T}'\mathbf{d}-\mathbf{dS}\right)\mathbf{A}+\mathbf{A}'\left(\mathbf{dT}-\mathbf{S}'\mathbf{d}\right)\mathbf{B}\right) \\
= & \mathbf{0},
\end{aligned}
$$

where the last equality follows from $\mathbf{T}'\mathbf{d}=\mathbf{dS}$ and $\mathbf{dT}=\mathbf{S}'\mathbf{d}$ (recall $\mathbf{T}_{in}q_i=\mathbf{S}_{ni}q_n$ and $\mathbf{d}\equiv diag\left(\boldsymbol{q}\right)$).

B.5 Proof of Proposition 5

The fact that $\mathbf{H}_{f_i}$ is real and symmetric, $\mu^{\max,i}$ is the largest eigenvalue and $\tilde{\mathbf{z}}^{\max,i}$ is the corresponding eigenvector implies that

$$
\left|\mu^{\max,i}\right|\equiv\max_{\mathbf{z}}\frac{\mathbf{z}^{\mathbf{T}}\mathbf{H}_{f_i}\mathbf{z}}{\mathbf{z}^{\mathbf{T}}\mathbf{z}},\quad \tilde{\mathbf{z}}^{\max,i}\equiv\arg\max_{\mathbf{z}}\frac{\mathbf{z}^{\mathbf{T}}\mathbf{H}_{f_i}\mathbf{z}}{\mathbf{z}^{\mathbf{T}}\mathbf{z}}.
$$

B.6 Proof of Proposition 6

The fact that $\mu^{\mathcal{A}}$ is the spectral norm implies that for all $\mathbf{z}$,

$$\mu^{\mathcal{A}}\|\mathbf{z}\|_2^4 \geq g(\mathbf{z}) = \frac{1}{N}\sum_{i=1}^{N}\left(\mathbf{z}'\mathbf{H}_{f_i}\mathbf{z}\right)^2 = \frac{1}{N}\sum_{i=1}^{N}\epsilon_i^2(\mathbf{z})$$

Hence $\sqrt{\frac{1}{N}\sum_{i=1}^{N}\epsilon_i^2(\mathbf{z})} \leq \sqrt{\mu^{\mathcal{A}}}\|\mathbf{z}\|_2^2$, as desired.

B.7 Proof of Proposition 10

To economize on notation we let $\mathbf{x} \equiv \mathrm{d}\ln\mathbf{w}$ and $\mathbf{y} \equiv \mathrm{d}\ln\mathbf{z}$, and we derive the sensitivity of $\mathbf{x}$ to ϵ, holding $\mathbf{y}$ fixed. Using our assumption of constant returns to scale in production, it is without loss of generality to normalize the weighted mean of productivity shocks $\mathbf{q}'\mathbf{y} = 0$ and thus $\mathbf{Q}\mathbf{y} = \mathbf{0}$. Note $(\mathbf{I}-\mathbf{V})\mathbf{x} = -\frac{\theta}{\theta+1}\mathbf{M}\mathbf{y}$, $\mathbf{W} = -\frac{\theta}{\theta+1}(\mathbf{I}-\mathbf{V})^{-1}\mathbf{M}$, $\mathbf{x} = (\mathbf{W}+\mathbf{Q})\mathbf{y}$, and $(\mathbf{W}+\mathbf{Q})$ is invertible. Differentiating and noting $\mathrm{d}\mathbf{V} = \mathrm{d}\mathbf{M}\,(=\epsilon\mathbf{O})$, we obtain

$$-\mathrm{d}\mathbf{V}\,\mathbf{x} + (\mathbf{I}-\mathbf{V})\,\mathrm{d}\mathbf{x} = -\frac{\theta}{\theta+1}\mathrm{d}\mathbf{M}\,\mathbf{y}$$

$$\implies \mathrm{d}\mathbf{x} = (\mathbf{I}-\mathbf{V})^{-1}\mathrm{d}\mathbf{V}(\mathbf{x}-\mathbf{y})$$

$$\implies \mathrm{d}\mathbf{x} = (\mathbf{I}-\mathbf{V})^{-1}\mathrm{d}\mathbf{V}\left(\mathbf{I}-(\mathbf{W}+\mathbf{Q})^{-1}\right)\mathbf{x}$$

By the Cauchy-Schwarz inequality,

$$\|\mathrm{d}\mathbf{x}\| \leq \|(\mathbf{I}-\mathbf{V})^{-1}\|\|\mathrm{d}\mathbf{V}\|\|\mathbf{I}-(\mathbf{W}+\mathbf{Q})^{-1}\|\|\mathbf{x}\|.$$

We obtain the proposition by substituting $\|\mathrm{d}\mathbf{V}\| = \frac{\theta}{\theta+1}\epsilon$ and $\|\mathrm{d}\mathbf{x}\|/\|\mathbf{x}\| = \lim_{\epsilon\to 0}\frac{\|\widetilde{\mathrm{d}\ln\mathbf{w}}-\mathrm{d}\ln\mathbf{w}\|}{\|\mathrm{d}\ln\mathbf{w}\|}$.

C General Armington

In this Section of the online appendix, we consider the Armington model with a general homothetic utility function in which goods are differentiated by country of origin from Section 2 of the paper. We consider a world consisting of many countries indexed by $i, n \in \{1, \ldots, N\}$. Each country has an exogenous supply of L_n workers, who are endowed with one unit of labor that is supplied inelastically.

C.1 Consumer Preferences

The preferences of the representative consumer in country n are characterized by the following homothetic indirect utility function:

$$u_n = \frac{w_n}{\mathcal{P}(\boldsymbol{p}_n)}, \tag{C.1}$$

where $\boldsymbol{p}_n$ is the vector of prices in country n of the goods produced by each country i with elements $p_{ni} = \tau_{ni}w_i/z_i$; w_n is the income of the representative consumer in country n; and $\mathcal{P}(\boldsymbol{p}_n)$ is a continuous and twice differentiable function that captures the ideal price index. From Roy's Identity, country n's demand for the good produced by country i is:

$$c_{ni} = c_{ni}(\boldsymbol{p}_n) = -\frac{\partial(1/\mathcal{P}(\boldsymbol{p}_n))}{\partial p_{ni}}w_n\mathcal{P}(\boldsymbol{p}_n). \tag{C.2}$$

C.2 Expenditure Shares

Country n's expenditure share on the good produced by country i is:

$$s_{ni} = \frac{p_{ni} c_{ni}\left(\boldsymbol{p}_n\right)}{\sum_{\ell=1}^{N} p_{n\ell} c_{n\ell}\left(\boldsymbol{p}_n\right)} \equiv \frac{x_{ni}\left(\boldsymbol{p}_n\right)}{\sum_{\ell=1}^{N} x_{n\ell}\left(\boldsymbol{p}_n\right)}. \tag{C.3}$$

Totally differentiating this expenditure share equation, we get:

$$\mathrm{d}s_{ni} = \frac{\mathrm{d}x_{ni}\left(\boldsymbol{p}_n\right)}{\sum_{\ell=1}^{N} x_{n\ell}\left(\boldsymbol{p}_n\right)} - \frac{x_{ni}\left(\boldsymbol{p}_n\right)}{\left[\sum_{\ell=1}^{N} x_{n\ell}\left(\boldsymbol{p}_n\right)\right]^2}\left[\sum_{k=1}^{N} \mathrm{d}x_{nk}\left(\boldsymbol{p}_n\right)\right],$$

$$\mathrm{d}s_{ni} = \frac{1}{\sum_{\ell=1}^{N} x_{n\ell}\left(\boldsymbol{p}_n\right)} \sum_{h=1}^{N} \frac{\partial x_{ni}\left(\boldsymbol{p}_n\right)}{\partial p_{nh}} \mathrm{d}p_{nh} - \frac{x_{ni}\left(\boldsymbol{p}_n\right)}{\left[\sum_{\ell=1}^{N} x_{n\ell}\left(\boldsymbol{p}_n\right)\right]^2} \sum_{k=1}^{N}\sum_{h=1}^{N} \frac{\partial x_{nk}\left(\boldsymbol{p}_n\right)}{\partial p_{nh}} \mathrm{d}p_{nh},$$

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = \sum_{h=1}^{N}\left[\left(\frac{\partial x_{ni}\left(\boldsymbol{p}_n\right)}{\partial p_{nh}}\frac{p_{nh}}{x_{ni}}\right) - \sum_{k=1}^{N} s_{nk}\left(\frac{\partial x_{nk}\left(\boldsymbol{p}_n\right)}{\partial p_{nh}}\frac{p_{nh}}{x_{nk}}\right)\right]\frac{\mathrm{d}p_{nh}}{p_{nh}},$$

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = \sum_{h=1}^{N}\left[\theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nkh}\right]\frac{\mathrm{d}p_{nh}}{p_{nh}}, \tag{C.4}$$

$$\mathrm{d}\ln s_{ni} = \sum_{h=1}^{N}\left[\theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nkh}\right]\mathrm{d}\ln p_{nh},$$

where

$$\theta_{nih} \equiv \left(\frac{\partial x_{ni}\left(\boldsymbol{p}_n\right)}{\partial p_{nh}}\frac{p_{nh}}{x_{ni}}\right) = \frac{\partial \ln x_{ni}}{\partial \ln p_{nh}}.$$

Totally differentiating prices in equation (3) in the paper, we have:

$$\frac{\mathrm{d}p_{ni}}{p_{ni}} = \frac{\mathrm{d}\tau_{ni}}{\tau_{ni}} + \frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i}{z_i}, \tag{C.5}$$

$$\mathrm{d}\ln p_{ni} = \mathrm{d}\ln\tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i.$$

C.3 Market Clearing

Market clearing implies:

$$w_i\ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \tag{C.6}$$

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i} w_i\ell_i = \sum_{n=1}^{N} \frac{\mathrm{d}s_{ni}}{s_{ni}} s_{ni} w_n\ell_n + \sum_{n=1}^{N} s_{ni}\frac{\mathrm{d}w_n}{w_n} w_n\ell_n,$$

$$\frac{\mathrm{d}w_i}{w_i} w_i\ell_i = \sum_{n=1}^{N} s_{ni} w_n\ell_n\left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}}\right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} \frac{s_{ni} w_n\ell_n}{w_i\ell_i}\left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}}\right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} t_{in}\left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}}\right),$$

where we have defined t_{in} as the share of country i's income from market n:

$$t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}.$$

Using our result for the derivatives of expenditure shares (C.4), we have:

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} t_{in} \left(\frac{\mathrm{d}w_n}{w_n} + \left[\sum_{h=1}^{N} \left[\theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nkh} \right] \frac{\mathrm{d}p_{nh}}{p_{nh}} \right] \right).$$

Using our results for the derivatives of prices, we have:

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} t_{in} \left(\frac{\mathrm{d}w_n}{w_n} + \left[\sum_{h=1}^{N} \left[\theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nkh} \right] \left[\frac{\mathrm{d}\tau_{nh}}{\tau_{nh}} + \frac{\mathrm{d}w_h}{w_h} - \frac{\mathrm{d}z_h}{z_h} \right] \right] \right),$$

which can be written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + \left[\sum_{h=1}^{N} \left[\theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nkh} \right] \left[\mathrm{d}\ln \tau_{nh} + \mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right] \right] \right), \tag{C.7}$$

which corresponds to equation (8) in the paper.

C.4 Welfare

Totally differentiating welfare, we have:

$$\mathrm{d}u_n = \mathrm{d}w_n \left(1/\mathcal{P}\left(\boldsymbol{p}_n\right)\right) + \mathrm{d}\left(1/\mathcal{P}\left(\boldsymbol{p}_n\right)\right) w_n,$$

$$\mathrm{d}u_n = \mathrm{d}w_n \left(1/\mathcal{P}\left(\boldsymbol{p}_n\right)\right) + \sum_{i=1}^{N} w_n \frac{\partial \left(1/\mathcal{P}\left(\boldsymbol{p}_n\right)\right)}{\partial p_{ni}} \mathrm{d}p_{ni},$$

$$\mathrm{d}u_n = \frac{\mathrm{d}w_n}{w_n} w_n \left(1/\mathcal{P}\left(\boldsymbol{p}_n\right)\right) + \sum_{i=1}^{N} w_n \frac{\partial \left(1/\mathcal{P}\left(\boldsymbol{p}_n\right)\right)}{\partial p_{ni}} p_{ni} \frac{\mathrm{d}p_{ni}}{p_{ni}},$$

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} + \sum_{i=1}^{N} \frac{\partial \left(1/\mathcal{P}\left(\boldsymbol{p}_n\right)\right)}{\partial p_{ni}} \mathcal{P}\left(\boldsymbol{p}_n\right) p_{ni} \frac{\mathrm{d}p_{ni}}{p_{ni}}.$$

Now recall from equation (C.2) that:

$$c_{ni} = -\frac{\partial \left(1/\mathcal{P}\left(\boldsymbol{p}_n\right)\right)}{\partial p_{ni}} w_n \mathcal{P}\left(\boldsymbol{p}_n\right).$$

Using this result in our total derivative for welfare above, we obtain:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{i=1}^{N} \frac{c_{ni}}{w_n} p_{ni} \frac{\mathrm{d}p_{ni}}{p_{ni}}.$$

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{i=1}^{N} s_{ni} \frac{\mathrm{d}p_{ni}}{p_{ni}}.$$

which can be equivalently written as:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni} \left[\mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right], \tag{C.8}$$

which corresponds to equation (10) in the paper.

D Constant Elasticity Armington

In this section of the online appendix, we report the derivations for our baseline constant elasticity of substitution Armington model in Section 3 of the paper. This model falls within the class of of quantitative trade models considered by Arkolakis, Costinot and Rodriguez-Clare (2012), which satisfy the three macro restrictions of (i) a constant elasticity import demand system, (ii) profits are a constant share of income, and (iii) trade is balanced. In Section E of this online appendix, we show that our analysis also holds for other models within this class, including those of perfect competition and constant returns to scale with Ricardian technology differences as in Eaton and Kortum (2002), and those of monopolistic competition and increasing returns to scale, in which goods are differentiated by firm, as in Krugman (1980) and Melitz (2003) with an untruncated Pareto productivity distribution. The world economy consists of many countries indexed by $i, n \in \{1, \ldots, N\}$. Each country n has an exogenous supply of labor ℓ_n.

D.1 Consumer Preferences

The preferences of the representative consumer in country n are characterized by the following indirect utility function:

$$u_n = \frac{w_n}{p_n}, \qquad p_n = \left[\sum_{i=1}^{N} p_{ni}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1, \tag{D.1}$$

where w_n is the wage; p_n is the consumption goods price index; p_{ni} is the price in country n of the good produced by country i; and we focus on the case in which countries' goods are substitutes ($\sigma > 1$).

D.2 Production Technology

Goods are produced with labor according to a constant returns to scale production technology. These goods can be traded between countries subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from country i to country n in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Therefore, the consumer in country n of purchasing a good ϑ from country i is:

$$p_{ni} = \frac{\tau_{ni} w_i}{z_i}, \tag{D.2}$$

where z_i captures productivity in country i and iceberg variable trade costs satisfy $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$.

D.3 Expenditure Shares

Using the properties of the CES demand function, country n's share of expenditure on goods produced in country i is:

$$s_{ni} = \frac{p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} p_{nm}^{1-\sigma}}. \tag{D.3}$$

Totally differentiating this expenditure share equation (D.3), we get:

$$\mathrm{d}s_{ni} = -\left(\sigma - 1\right) \frac{\frac{d\,\mathrm{d}_{ni}}{p_{ni}} p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} p_{nm}^{1-\sigma}} + \left(\sigma - 1\right) \sum_{h=1}^{N} \frac{p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} p_{nm}^{1-\sigma}} \frac{\frac{\mathrm{d}p_{nh}}{p_{nh}} p_{nh}^{1-\sigma}}{\sum_{m=1}^{N} p_{nm}^{1-\sigma}},$$

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = -\left(\sigma - 1\right) \frac{\mathrm{d}p_{ni}}{p_{ni}} + \left(\sigma - 1\right) \sum_{h=1}^{N} \frac{\frac{\mathrm{d}p_{nh}}{p_{nh}} p_{nh}^{1-\sigma}}{\sum_{m=1}^{N} p_{nm}^{1-\sigma}},$$

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = -(\sigma-1)\frac{\mathrm{d}p_{ni}}{p_{ni}} + (\sigma-1)\sum_{h=1}^{N} s_{nh}\frac{\mathrm{d}p_{nh}}{p_{nh}},$$

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = (\sigma-1)\left(\sum_{h=1}^{N} s_{nh}\frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}}\right), \tag{D.4}$$

$$\mathrm{d}\ln s_{ni} = (\sigma-1)\left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni}\right),$$

where, from the definition of p_{ni} in equation (D.2) above, we have:

$$\frac{\mathrm{d}p_{ni}}{p_{ni}} = \frac{\mathrm{d}\tau_{ni}}{\tau_{ni}} + \frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i}{z_i}, \tag{D.5}$$

$$\mathrm{d}\ln p_{ni} = \mathrm{d}\ln\tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i.$$

D.4 Price Indices

Totally differentiating the consumption goods price index in equation (D.1), we have:

$$\mathrm{d}p_n = \sum_{m=1}^{N}\frac{\mathrm{d}p_{nm}}{p_{nm}}\frac{p_{nm}^{1-\sigma}}{\sum_{h=1}^{N}p_{nh}^{1-\sigma}}\left[\sum_{m=1}^{N}p_{nm}^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

$$\frac{\mathrm{d}p_n}{p_n} = \sum_{m=1}^{N}\frac{\mathrm{d}p_{nm}}{p_{nm}}\frac{p_{nm}^{1-\sigma}}{\sum_{h=1}^{N}p_{nh}^{1-\sigma}},$$

$$\frac{\mathrm{d}p_n}{p_n} = \sum_{m=1}^{N} s_{nm}\frac{\mathrm{d}p_{nm}}{p_{nm}}, \tag{D.6}$$

$$\mathrm{d}\ln p_n = \sum_{m=1}^{N} s_{nm}\,\mathrm{d}\ln p_{nm}.$$

D.5 Market Clearing

Market clearing requires that income in each country equals expenditure on the goods produced in that country:

$$w_i\ell_i = \sum_{n=1}^{N} s_{ni}w_n\ell_n. \tag{D.7}$$

Totally differentiating this market clearing condition (D.7), holding labor endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i}w_i\ell_i = \sum_{n=1}^{N}\frac{\mathrm{d}s_{ni}}{s_{ni}}s_{ni}w_n\ell_n + \sum_{n=1}^{N}s_{ni}\frac{\mathrm{d}w_n}{w_n}w_n\ell_n,$$

$$\frac{\mathrm{d}w_i}{w_i}w_i\ell_i = \sum_{n=1}^{N}s_{ni}w_n\ell_n\left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}}\right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N}\frac{s_{ni}w_n\ell_n}{w_i\ell_i}\left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}}\right).$$

Using our result for the derivative of expenditure shares in equation (D.4) above, we can rewrite this as:

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N}\frac{s_{ni}w_n\ell_n}{w_i\ell_i}\left(\frac{\mathrm{d}w_n}{w_n} + (\sigma-1)\left(\sum_{h=1}^{N}s_{nh}\frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}}\right)\right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} t_{in} \left(\frac{\mathrm{d}w_n}{w_n} + (\sigma - 1) \left(\sum_{h \in N} s_{nh} \frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}} \right) \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh}\, \mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni} \right) \right), \tag{D.8}$$

where we have defined t_{in} as the share of country i's income derived from market n:

$$t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}.$$

D.6 Utility Again

Returning to our expression for indirect utility, we have:

$$u_n = \frac{w_n}{p_n}. \tag{D.9}$$

Totally differentiating indirect utility (D.9), we have:

$$\mathrm{d}u_n = \frac{\frac{\mathrm{d}w_n}{w_n} w_n}{p_n} - \frac{\frac{\mathrm{d}p_n}{p_n} w_n}{p_n},$$

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \frac{\mathrm{d}p_n}{p_n}.$$

Using our total derivative of the sectoral price index in equation (D.6) above, we get:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{m=1}^{N} s_{nm} \frac{\mathrm{d}p_{nm}}{p_{nm}}, \tag{D.10}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\, \mathrm{d}\ln p_{nm}.$$

D.7 Wages and Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

$$\mathrm{d}\ln \tau_{ni} \quad = 0, \quad \forall\, n, i \in N. \tag{D.11}$$

We start with our expression for the log change in wages from equation (D.8) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh}\, \mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni} \right) \right),$$

Using the total derivative of prices (D.5) and our assumption of constant bilateral trade costs (D.11), we can write this expression for the log change in wages as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right] \right) \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\, \mathrm{d}\ln w_n + (\sigma - 1) \sum_{n=1}^{N} t_{in} \left(\sum_{h=1}^{N} s_{nh} \left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right] \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\, \mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} s_{nh} \left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right] \right), \tag{D.12}$$

which can be re-written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\left(\sum_{n=1}^{N} m_{in}\left[\mathrm{d}\ln w_n - \mathrm{d}\ln z_n\right]\right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih}s_{hn} - 1_{n=i},$$

which has the following matrix representation in the paper:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\,\mathrm{d}\ln \mathbf{w} + \theta\mathbf{M}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right), \tag{D.13}$$

$$\theta = \sigma - 1.$$

We solve for our friend-enemy income exposure measure by matrix inversion. Dividing both sides of equation (D.13) by $\theta+1$, we have:

$$\frac{1}{\theta+1}\,\mathrm{d}\ln \mathbf{w} = \frac{1}{\theta+1}\mathbf{T}\,\mathrm{d}\ln \mathbf{w} + \frac{\theta}{\theta+1}\mathbf{M}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right),$$

$$\frac{1}{\theta+1}\left(\mathbf{I} - \mathbf{T} - \theta\mathbf{M}\right)\,\mathrm{d}\ln \mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln \mathbf{z}.$$

Now using $\mathbf{M} = \mathbf{TS} - \mathbf{I}$, we have:

$$\frac{1}{\theta+1}\left(\mathbf{I} - \mathbf{T} - \theta\mathbf{TS} + \theta\mathbf{I}\right)\,\mathrm{d}\ln \mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln \mathbf{z},$$

$$\left(\mathbf{I} - \frac{\mathbf{T} + \theta\mathbf{TS}}{\theta+1}\right)\,\mathrm{d}\ln \mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln \mathbf{z}.$$

Using our choice of world GDP as numeraire, which implies $\mathbf{Q}\,\mathrm{d}\ln \mathbf{w} = 0$, we have:

$$\left(\mathbf{I} - \frac{\mathbf{T} + \theta\mathbf{TS}}{\theta+1} + \mathbf{Q}\right)\,\mathrm{d}\ln \mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln \mathbf{z},$$

which can be re-written as:

$$\left(\mathbf{I} - \mathbf{V}\right)\,\mathrm{d}\ln \mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln \mathbf{z},$$

$$\mathbf{V} \equiv \frac{\mathbf{T} + \theta\mathbf{TS}}{\theta+1} - \mathbf{Q},$$

which yields the following solution of the change in wages in response to a productivity shock:

$$\mathrm{d}\ln \mathbf{w} = -\frac{\theta}{\theta+1}\left(\mathbf{I} - \mathbf{V}\right)^{-1}\mathbf{M}\,\mathrm{d}\ln \mathbf{z},$$

which can be re-written as:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{W}\,\mathrm{d}\ln \mathbf{z},$$

where $\mathbf{W}$ is our friend-enemy income exposure measure:

$$\mathbf{W} \equiv -\frac{\theta}{\theta+1}\left(\mathbf{I} - \mathbf{V}\right)^{-1}\mathbf{M}. \tag{D.14}$$

D.8 Welfare and Productivity Shocks

From equation (D.10), the log change in utility is given by:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\,\mathrm{d}\ln p_{nm},$$

Using the total derivative of prices (D.5) and our assumption of constant bilateral trade costs (D.11), we can write this log change in utility as:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\left[\mathrm{d}\ln w_m - \mathrm{d}\ln z_m\right], \tag{D.15}$$

which has the following matrix representation in the paper:

$$\mathrm{d}\ln \mathbf{u} = \mathrm{d}\ln \mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right). \tag{D.16}$$

We can re-write the above relationship as:

$$\mathrm{d}\ln \mathbf{u} = \left(\mathbf{I} - \mathbf{S}\right)\,\mathrm{d}\ln \mathbf{w} + \mathbf{S}\,\mathrm{d}\ln \mathbf{z},$$

which using our solution for $\mathrm{d}\ln \mathbf{w}$ from above, can be further re-written as:

$$\mathrm{d}\ln \mathbf{u} = \left(\mathbf{I} - \mathbf{S}\right)\mathbf{W}\,\mathrm{d}\ln \mathbf{z} + \mathbf{S}\,\mathrm{d}\ln \mathbf{z},$$

$$\mathrm{d}\ln \mathbf{u} = \left[\left(\mathbf{I} - \mathbf{S}\right)\mathbf{W} + \mathbf{S}\right]\,\mathrm{d}\ln \mathbf{z},$$

$$\mathrm{d}\ln \mathbf{u} = \mathbf{U}\,\mathrm{d}\ln \mathbf{z},$$

where $\mathbf{U}$ is our friend-enemy welfare exposure measure:

$$\mathbf{U} \equiv \left[\left(\mathbf{I} - \mathbf{S}\right)\mathbf{W} + \mathbf{S}\right]. \tag{D.17}$$

D.9 Market-size and Cross-Substitution Effects

We define the cross-substitution effect as the wage vector that solves equation (D.13) replacing the term $\mathbf{T}\,\mathrm{d}\ln \mathbf{w}$ with $\mathbf{Q}\,\mathrm{d}\ln \mathbf{w}$:

$$\mathrm{d}\ln \mathbf{w}^{\text{Sub}} = \mathbf{Q}\,\mathrm{d}\ln \mathbf{w}^{\text{Sub}} + \theta\cdot\mathbf{M}\left(\mathrm{d}\ln \mathbf{w}^{\text{Sub}} - \mathrm{d}\ln \mathbf{z}\right),$$

where we use the superscript Sub to indicate the cross-substitution effect.

We solve for our friend-enemy measure of income exposure due to the cross-substitution effect using matrix inversion. Dividing both sides of the above equation by $\theta+1$, we have:

$$\frac{1}{\theta+1}\,\mathrm{d}\ln \mathbf{w}^{\text{Sub}} = \frac{1}{\theta+1}\mathbf{Q}\,\mathrm{d}\ln \mathbf{w}^{\text{Sub}} + \frac{\theta}{\theta+1}\mathbf{M}\left(\mathrm{d}\ln \mathbf{w}^{\text{Sub}} - \mathrm{d}\ln \mathbf{z}\right),$$

$$\frac{1}{\theta+1}\left(\mathbf{I} - \mathbf{Q} - \theta\mathbf{M}\right)\,\mathrm{d}\ln \mathbf{w}^{\text{Sub}} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln \mathbf{z}.$$

Now using $\mathbf{M} = \mathbf{TS} - \mathbf{I}$, we have:

$$\frac{1}{\theta+1}\left(\mathbf{I} - \mathbf{Q} - \theta\mathbf{TS} + \theta\mathbf{I}\right)\,\mathrm{d}\ln \mathbf{w}^{\text{Sub}} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln \mathbf{z},$$

$$\left(\mathbf{I} - \frac{\mathbf{Q} + \theta\mathbf{TS}}{\theta+1}\right)\,\mathrm{d}\ln \mathbf{w}^{\text{Sub}} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln \mathbf{z}.$$

Using our choice of world GDP as numeraire, which implies $\mathbf{Q}\,\mathrm{d}\mathbf{w}^{\text{Sub}} = 0$, we have:

$$\left(\mathbf{I} - \frac{\mathbf{Q} + \theta\mathbf{TS}}{\theta+1} + \mathbf{Q}\right)\,\mathrm{d}\ln\mathbf{w}^{\text{Sub}} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

$$\left(\mathbf{I} - \frac{\theta\mathbf{Q} + \theta\mathbf{TS}}{\theta+1}\right)\,\mathrm{d}\ln\mathbf{w}^{\text{Sub}} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z}.$$

Now using $\mathbf{M} = \mathbf{TS} - \mathbf{I}$, we have:

$$\left(\mathbf{I} - \frac{\theta\mathbf{Q} + \theta\mathbf{M}}{\theta+1} - \frac{\theta}{\theta+1}\mathbf{I}\right)\,\mathrm{d}\ln\mathbf{w}^{\text{Sub}} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

$$\left(\frac{\mathbf{I}}{\theta+1} - \frac{\theta\mathbf{Q} + \theta\mathbf{M}}{\theta+1}\right)\,\mathrm{d}\ln\mathbf{w}^{\text{Sub}} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

$$\left(\mathbf{I} - (\theta\mathbf{Q} + \theta\mathbf{M})\right)\,\mathrm{d}\ln\mathbf{w}^{\text{Sub}} = -\theta\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

which yields the following solution of the change in wages in response to a productivity shock:

$$\mathrm{d}\ln\mathbf{w}^{\text{Sub}} = -\theta\left(\mathbf{I} - (\theta\mathbf{Q} + \theta\mathbf{M})\right)^{-1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

which can be re-written as:

$$\mathrm{d}\ln\mathbf{w}^{\text{Sub}} = \mathbf{W}^{\text{Sub}}\,\mathrm{d}\ln\mathbf{z},$$

where $\mathbf{W}^{\text{Sub}}$ is our friend-enemy measure of income exposure due to the cross-substitution effect:

$$\mathbf{W}^{\text{Sub}} \equiv -\theta\left(\mathbf{I} - (\theta\mathbf{Q} + \theta\mathbf{M})\right)^{-1}\mathbf{M}.$$

D.10 Relationship to the ACR Gains from Trade Formula

From equation (D.10), the log change in utility is given by:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\,\mathrm{d}\ln p_{nm}. \tag{D.18}$$

Choosing country n's wage as the numeraire and assuming no changes in its productivity or domestic trade costs, we have:

$$\mathrm{d}\ln w_n = 0, \quad \mathrm{d}\ln z_n = 0, \quad \mathrm{d}\ln\tau_{nn} = 0, \quad \mathrm{d}\ln p_{nn} = 0.$$

The import demand system in equation (D.3) implies:

$$\mathrm{d}\ln s_{nm} - \mathrm{d}\ln s_{nn} = -(\sigma-1)\left(\mathrm{d}\ln p_{nm} - \mathrm{d}\ln p_{nn}\right).$$

Using this result in equation (D.18), the log change in welfare can be written as:

$$\mathrm{d}\ln u_n = \sum_{m=1}^{N} \frac{s_{nm}\left(\mathrm{d}\ln s_{nm} - \mathrm{d}\ln s_{nn}\right)}{(\sigma-1)}$$

Using $\sum_{m=1}^{N} s_{nm} = 1$ and $\sum_{m=1}^{N} \mathrm{d}s_{nm} = 0$, we obtain the ACR welfare gains from trade formula for small changes:

$$\mathrm{d}\ln u_n = -\frac{\mathrm{d}\ln s_{nn}}{\sigma-1}. \tag{D.19}$$

Integrating both sides of equation (D.19), we get:

$$\int_{u_n^0}^{u_n^1} \frac{\mathrm{d}u_n}{u_n} = -(\sigma-1)\int_{s_{nn}^0}^{s_{nn}^1} \frac{\mathrm{d}s_{nn}}{s_{nn}},$$

$$\ln\left(\frac{u_n^1}{u_n^0}\right) = -(\sigma-1)\ln\left(\frac{s_{nn}^1}{s_{nn}^0}\right),$$

$$\left(\frac{s_n^1}{s_n^0}\right) = \left(\frac{s_{nn}^1}{s_{nn}^0}\right)^{-(\sigma-1)}, \tag{D.20}$$

which corresponds to the ACR welfare gains from trade formula for large changes.

E Single-Sector Isomorphisms

While for convenience of exposition we focus on the single-sector Armington model in the paper, we obtain the same income and welfare exposure measures for all models in the ACR class with a constant trade elasticity. In Subsection E.1, we derive our exposure measures in a version of the Eaton and Kortum (2002) model. In Subsection E.2, we derive these measures in the Krugman (1980) model.

The Armington model in the paper and the Eaton and Kortum (2002) model assume perfect competition and constant returns to scale, whereas the Krugman (1980) model assumes monopolistic competition and increasing returns to scale. In the Eaton and Kortum (2002) model, the trade elasticity corresponds to the Fréchet shape parameter. In contrast, in the Armington model and the Krugman (1980) model, the trade elasticity corresponds to the elasticity of substitution between varieties. Nevertheless, our income and welfare exposure measures hold across all three models, because they all feature a constant trade elasticity.

In Subsection E.2, we focus on the representative firm Krugman (1980) model of monopolistic competition for simplicity, but our income and welfare exposure measures also hold in the heterogeneous firm model of Melitz (2003) with an untruncated Pareto productivity distribution. In Subsection E.2, we also show that our income and welfare exposure measures with monopolistic competition and increasing returns to scale take a similar form whether we consider shocks to the variable or fixed components of production costs.

E.1 Eaton and Kortum (2002)

We consider a version of Eaton and Kortum (2002) with labor as the sole factor of production. Trade arises because of Ricardian technology differences; production technologies are constant returns to scale; and markets are perfectly competitive. The world economy consists of a set of countries indexed by $i, n \in \{1, \ldots, N\}$. Each country n has an exogenous supply of labor ℓ_n.

E.1.1 Consumer Preferences

The preferences of the representative consumer in country n are characterized by the following indirect utility function:

$$u_n = \frac{w_n}{p_n}, \tag{E.1}$$

where w_n is the wage and p_n is the consumption goods price index, which is defined over consumption of a fixed continuum of goods according to the constant elasticity of substitution (CES) functional form:

$$p_n = \left[\int_0^1 p_n(\vartheta)^{1-\sigma}\, d\vartheta\right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1, \tag{E.2}$$

where $p_n(\vartheta)$ denotes the price of good ϑ in country n.

E.1.2 Production Technology

Goods are produced with labor according to a constant returns to scale production technology. These goods can be traded between countries subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from country i to country n in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Therefore, the price for consumers in country n of purchasing a good ϑ from country i is:

$$p_{ni}(\vartheta) = \frac{\tau_{ni} w_i}{z_i a_i(\vartheta)}, \tag{E.3}$$

where z_i captures common determinants of productivity across goods within country i and $a_i(\vartheta)$ captures idiosyncratic determinants of productivity for each good ϑ within that country. Iceberg variable trade costs satisfy $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$. Productivity for each good ϑ in each sector k and each country i is drawn independently from the following Fréchet distribution:

$$F_i(a) = \exp\left(-a^{-\theta}\right), \qquad \theta > 1, \tag{E.4}$$

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to z_i.

E.1.3 Expenditure Shares

Using the properties of this Fréchet distribution, country n's share of expenditure on goods produced in country i is:

$$s_{ni} = \frac{\left(\tau_{ni} w_i / z_i\right)^{-\theta}}{\sum_{m=1}^{N} \left(\tau_{nm} w_m / z_m\right)^{-\theta}} = \frac{\left(\rho_{ni}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\rho_{nm}\right)^{-\theta}}, \tag{E.5}$$

where we have defined the following price inclusive of trade costs term:

$$\rho_{ni} \equiv \frac{\tau_{ni} w_i}{z_i}. \tag{E.6}$$

Totally differentiating this expenditure share equation (E.5) we get:

$$\mathrm{d}s_{ni} = -\frac{\theta \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} \left(\rho_{ni}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\rho_{nm}\right)^{-\theta}} + \sum_{h=1}^{N} \frac{\left(\rho_{ni}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\rho_{nm}\right)^{-\theta}} \frac{\theta \frac{\mathrm{d}\rho_{nh}}{\rho_{nh}} \left(\rho_{nh}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\rho_{nm}\right)^{-\theta}},$$

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = -\theta \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} + \sum_{h=1}^{N} \frac{\theta \frac{\mathrm{d}\rho_{nh}}{\rho_{nh}} \left(\rho_{nh}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\rho_{nm}\right)^{-\theta}},$$

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = -\theta \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} + \sum_{h=1}^{N} s_{nh} \theta \frac{\mathrm{d}\rho_{nh}}{\rho_{nh}},$$

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = \theta \left(\sum_{h=1}^{N} s_{nh} \frac{\mathrm{d}\rho_{nh}}{\rho_{nh}} - \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} \right), \tag{E.7}$$

$$\mathrm{d}\ln s_{ni} = \theta \left(\sum_{h=1}^{N} s_{nh} \, \mathrm{d}\ln \rho_{nh} - \mathrm{d}\ln \rho_{ni} \right),$$

where, from the definition of ρ_{ni} in equation (E.6) above, we have:

$$\frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} = \frac{\mathrm{d}\tau_{ni}}{\tau_{ni}} + \frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i}{z_i}, \tag{E.8}$$

$$\mathrm{d}\ln \rho_{ni} = \mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i.$$

E.1.4 Price Indices

Using the properties of the Fréchet distribution (E.4), the consumption goods price index is given by:

$$p_n = \gamma \left[\sum_{m=1}^{N} (\rho_{nm})^{-\theta} \right]^{-\frac{1}{\theta}}, \tag{E.9}$$

where

$$\gamma \equiv \left[\Gamma \left(\frac{\theta + 1 - \sigma}{\theta} \right) \right]^{\frac{1}{1-\sigma}},$$

and $\Gamma(\cdot)$ is the Gamma function. Totally differentiating this price index (E.9), we have:

$$\begin{aligned}
\mathrm{d}p_n &= \sum_{m=1}^{N} \gamma \frac{\mathrm{d}\rho_{nm}}{\rho_{nm}} \frac{(\rho_{nm})^{-\theta}}{\sum_{h=1}^{N} (\rho_{nh})^{-\theta}} \left[\sum_{m=1}^{N} (\rho_{nm})^{-\theta} \right]^{-\frac{1}{\theta}}, \\
\frac{\mathrm{d}p_n}{p_n} &= \sum_{m=1}^{N} \frac{\mathrm{d}\rho_{nm}}{\rho_{nm}} \frac{(\rho_{nm})^{-\theta}}{\sum_{h=1}^{N} (\rho_{nh})^{-\theta}}, \\
\frac{\mathrm{d}p_n}{p_n} &= \sum_{m=1}^{N} s_{nm} \frac{\mathrm{d}\rho_{nm}}{\rho_{nm}}, \qquad \text{(E.10)} \\
\mathrm{d} \ln p_n &= \sum_{m=1}^{N} s_{nm} \, \mathrm{d} \ln \rho_{nm}.
\end{aligned}$$

E.1.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

$$w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \tag{E.11}$$

Totally differentiating this market clearing condition (E.11), holding labor endowments constant, we have:

$$\begin{aligned}
\frac{\mathrm{d}w_i}{w_i} w_i \ell_i &= \sum_{n=1}^{N} \frac{\mathrm{d}s_{ni}}{s_{ni}} s_{ni} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \frac{\mathrm{d}w_n}{w_n} w_n \ell_n, \\
\frac{\mathrm{d}w_i}{w_i} w_i \ell_i &= \sum_{n=1}^{N} s_{ni} w_n \ell_n \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}} \right), \\
\frac{\mathrm{d}w_i}{w_i} &= \sum_{n=1}^{N} \frac{s_{ni} w_n \ell_n}{w_i \ell_i} \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}} \right).
\end{aligned}$$

Using our result for the derivative of expenditure shares in equation (E.7) above, we can rewrite this as:

$$\begin{aligned}
\frac{\mathrm{d}w_i}{w_i} &= \sum_{n=1}^{N} \frac{s_{ni} w_n \ell_n}{w_i \ell_i} \left(\frac{\mathrm{d}w_n}{w_n} + \theta \left(\sum_{h=1}^{N} s_{nh} \frac{\mathrm{d}\rho_{nh}}{\rho_{nh}} - \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} \right) \right), \\
\frac{\mathrm{d}w_i}{w_i} &= \sum_{n=1}^{N} t_{in} \left(\frac{\mathrm{d}w_n}{w_n} + \theta \left(\sum_{h \in N} s_{nh} \frac{\mathrm{d}\rho_{nh}}{\rho_{nh}} - \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} \right) \right), \\
\mathrm{d} \ln w_i &= \sum_{n=1}^{N} t_{in} \left(\mathrm{d} \ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh} \, \mathrm{d} \ln \rho_{nh} - \mathrm{d} \ln \rho_{ni} \right) \right), \qquad \text{(E.12)}
\end{aligned}$$

where we have defined t_{in} as country n's expenditure on country i as a share of country i's income:

$$t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}.$$

E.1.6 Utility Again

Returning to our expression for indirect utility, we have:

$$u_n = \frac{w_n}{p_n}. \tag{E.13}$$

Totally differentiating indirect utility (E.13), we have:

$$\mathrm{d}u_n = \frac{\frac{\mathrm{d}w_n}{w_n} w_n}{p_n} - \frac{\frac{\mathrm{d}p_n}{p_n} w_n}{p_n},$$

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \frac{\mathrm{d}p_n}{p_n}.$$

Using our total derivative of the sectoral price index in equation (E.10) above, we get:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{m=1}^{N} s_{nm} \frac{\mathrm{d}\rho_{nm}}{\rho_{nm}}, \tag{E.14}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\, \mathrm{d}\ln \rho_{nm}.$$

E.1.7 Wages and Common Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

$$\mathrm{d}\ln \tau_{ni} \quad = 0, \quad \forall\, n, i \in N. \tag{E.15}$$

We start with our expression for the log change in wages from equation (E.12) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}\, \mathrm{d}\ln \rho_{nh} - \mathrm{d}\ln \rho_{ni} \right) \right),$$

Using the total derivative of the price term (E.8) and our assumption of constant bilateral trade costs (E.15), we can write this expression for the log change in wages as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh} \left(\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right) - \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right) \right) \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\, \mathrm{d}\ln w_n + \theta \sum_{n=1}^{N} t_{in} \left(\sum_{h=1}^{N} s_{nh} \left(\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right) - \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right) \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\, \mathrm{d}\ln w_n + \theta \left(\sum_{h\in N}^{N} \sum_{n=1}^{N} t_{in} s_{nh} \left(\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right) - \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right) \right). \tag{E.16}$$

which can be re-written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\, \mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{n=1}^{N} m_{in} \left[\mathrm{d}\ln w_n - \mathrm{d}\ln z_n \right] \right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i},$$

which has the same matrix representation as in equation (15) in the paper:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\, \mathrm{d}\ln \mathbf{w} + \theta \mathbf{M} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right), \tag{E.17}$$

$$\theta = \sigma - 1.$$

E.1.8 Utility and Common Productivity Shocks

From equation (E.14), the log change in utility is given by:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\,\mathrm{d}\ln \rho_{nm}.$$

Using the total derivative of the price term (E.8) and our assumption of constant bilateral trade costs (E.15), we can write this log change in utility as:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\left[\mathrm{d}\ln w_m - \mathrm{d}\ln z_m\right],$$

which has the same matrix representation as in equation (19) in the paper:

$$\mathrm{d}\ln \mathbf{u} = \mathrm{d}\ln \mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right). \tag{E.18}$$

E.2 Krugman (1980)

We consider a version of Krugman (1980) with labor as the sole factor of production, in which markets are monopolistically competitive, and trade arises from love of variety and increasing returns to scale. The world economy consists of a set of countries indexed by $i, n \in \{1, \ldots, N\}$. Each country n has an exogenous supply of labor ℓ_n.

E.2.1 Consumer Preferences

The preferences of the representative consumer in country n are characterized by the following indirect utility function:

$$u_n = \frac{w_n}{p_n}, \qquad p_n = \left[\sum_{i=1}^{N} \int_0^{M_i} p_{ni}\left(j\right)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1, \tag{E.19}$$

where w_n is the wage; p_n is the consumption goods price index; $p_{ni}\left(j\right)$ is the price in country n of a variety j produced in country i; M_i is the endogenous mass of varieties; and varieties are substitutes ($\sigma > 1$).

E.2.2 Production Technology

Varieties are produced under conditions of monopolistic competition and increasing returns to scale. To produce a variety, a firm must incur a fixed cost of F units of labor and a constant variable cost in terms of labor that depends on a location's productivity z_i. Therefore the total amount of labor ($l_i\left(j\right)$) required to produce $x_i\left(j\right)$ units of variety j in location i is:

$$l_i\left(j\right) = F_i + \frac{x_i\left(j\right)}{z_i}. \tag{E.20}$$

Varieties can be traded between countries subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from country i to country n in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Profit maximization and zero profits imply that equilibrium prices are a constant markup over marginal cost:

$$p_{ni}\left(j\right) = \left(\frac{\sigma}{\sigma - 1}\right)\rho_{ni}, \qquad \rho_{ni} \equiv \frac{\tau_{ni} w_i}{z_i}, \tag{E.21}$$

and equilibrium employment for each variety is equal to a constant:

$$l_i\left(j\right) = \bar{l} = \left(\sigma - 1\right) F_i. \tag{E.22}$$

Given this constant equilibrium employment for each variety, labor market clearing implies that the total mass of varieties supplied by each country is proportional to its labor endowment:

$$M_i = \frac{\ell_i}{\sigma F_i}. \tag{E.23}$$

From the definition of ρ_{ni} in equation (E.21) above, we have:

$$\frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} = \frac{\mathrm{d}\tau_{ni}}{\tau_{ni}} + \frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i}{z_i}, \tag{E.24}$$
$$\mathrm{d}\ln \rho_{ni} = \mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i.$$

Totally differentiating in the labor market clearing condition (E.23), holding country endowments constant, we have:

$$\frac{\mathrm{d}M_i}{M_i} = -\frac{\mathrm{d}F_i}{F_i}, \tag{E.25}$$
$$\mathrm{d}\ln M_i = -\mathrm{d}\ln F_i.$$

E.2.3 Expenditure Shares

Using the symmetry of equilibrium prices and the properties of the CES demand function, country n's share of expenditure on goods produced in country i is:

$$s_{ni} = \frac{M_i p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} M_m p_{nm}^{1-\sigma}} = \frac{\left(\ell_i/F_i\right)\rho_{ni}^{1-\sigma}}{\sum_{m=1}^{N}\left(\ell_m/F_m\right)\rho_{nm}^{1-\sigma}}. \tag{E.26}$$

Totally differentiating this expenditure share equation (E.26) we get:

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = -\left[\frac{\mathrm{d}F_i}{F_i} + (\sigma-1)\frac{\mathrm{d}\rho_{ni}}{\rho_{ni}}\right] + \sum_{h=1}^{N} s_{nh}\left[\frac{\mathrm{d}F_h}{F_h} + (\sigma-1)\frac{\mathrm{d}\rho_{nh}}{\rho_{nh}}\right], \tag{E.27}$$

$$\mathrm{d}\ln s_{ni} = -\left[\mathrm{d}\ln F_i + (\sigma-1)\,\mathrm{d}\ln \rho_{ni}\right] + \sum_{h=1}^{N} s_{nh}\left[\mathrm{d}\ln F_h + (\sigma-1)\,\mathrm{d}\ln \rho_{nh}\right].$$

E.2.4 Price Indices

Using the symmetry of equilibrium prices, the price index (E.19) can be re-written as:

$$p_n = \left[\sum_{i=1}^{N} M_i p_{ni}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

Totally differentiating this price index, we have:

$$\mathrm{d}p_n = \sum_{i=1}^{N}\left[\frac{1}{1-\sigma}\frac{\mathrm{d}M_i}{M_i} + \frac{\mathrm{d}p_{ni}}{p_{ni}}\right]\frac{M_i p_{ni}^{1-\sigma}}{\sum_{h=1}^{N} M_h p_{nh}^{1-\sigma}}\left[\sum_{i=1}^{N} M_i p_{ni}^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

$$\frac{\mathrm{d}p_n}{p_n} = \sum_{i=1}^{N}\left[\frac{1}{1-\sigma}\frac{\mathrm{d}M_i}{M_i} + \frac{\mathrm{d}p_{ni}}{p_{ni}}\right]\frac{p_{ni}^{1-\sigma}}{\sum_{h=1}^{N} p_{nh}^{1-\sigma}},$$

$$\frac{\mathrm{d}p_n}{p_n} = \sum_{i=1}^{N} s_{ni}\left[\frac{1}{1-\sigma}\frac{\mathrm{d}M_i}{M_i} + \frac{\mathrm{d}p_{ni}}{p_{ni}}\right],$$

which using the equilibrium pricing rule (E.21) and the labor market clearing condition (E.23) can be written as:

$$\frac{\mathrm{d}p_n}{p_n} = \sum_{i=1}^{N} s_{ni}\left[\frac{1}{\sigma-1}\frac{\mathrm{d}F_i}{F_i} + \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}}\right], \tag{E.28}$$

$$\mathrm{d}\ln p_n = \sum_{i=1}^{N} s_{ni}\left[\frac{1}{\sigma-1}\mathrm{d}\ln F_i + \mathrm{d}\ln \rho_{ni}\right].$$

E.2.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

$$w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \tag{E.29}$$

Totally differentiating this market clearing condition (E.29), holding labor endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} \frac{\mathrm{d}s_{ni}}{s_{ni}} s_{ni} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \frac{\mathrm{d}w_n}{w_n} w_n \ell_n,$$

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}} \right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} \frac{s_{ni} w_n \ell_n}{w_i \ell_i} \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}} \right).$$

Using our result for the derivative of expenditure shares in equation (E.27) above, we can rewrite this as:

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} \frac{s_{ni} w_n \ell_n}{w_i \ell_i} \left(\begin{array}{c} \frac{\mathrm{d}w_n}{w_n} + \left(\sum_{h=1}^{N} s_{nh} \frac{\mathrm{d}F_h}{F_h} - \frac{\mathrm{d}F_i}{F_i} \right) \\ + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \frac{\mathrm{d}\rho_{nh}}{\rho_{nh}} - \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} \right) \end{array} \right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} t_{in} \left(\begin{array}{c} \frac{\mathrm{d}w_n}{w_n} + \left(\sum_{h \in N} s_{nh} \frac{\mathrm{d}F_h}{F_h} - \frac{\mathrm{d}F_i}{F_i} \right) \\ + (\sigma - 1) \left(\sum_{h \in N} s_{nh} \frac{\mathrm{d}\rho_{nh}}{\rho_{nh}} - \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} \right) \end{array} \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\begin{array}{c} \mathrm{d}\ln w_n + \left(\sum_{h=1}^{N} s_{nh} \,\mathrm{d}\ln F_h - \mathrm{d}\ln F_i \right) \\ + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \,\mathrm{d}\ln \rho_{nh} - \mathrm{d}\ln \rho_{ni} \right) \end{array} \right), \tag{E.30}$$

where we have defined t_{in} as the share of country i's income derived from market n:

$$t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}.$$

E.2.6 Utility Again

Returning to our expression for indirect utility, we have:

$$u_n = \frac{w_n}{p_n}. \tag{E.31}$$

Totally differentiating indirect utility (E.31), we have:

$$\mathrm{d}u_n = \frac{\frac{\mathrm{d}w_n}{w_n} w_n}{p_n} - \frac{\frac{\mathrm{d}p_n}{p_n} w_n}{p_n},$$

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \frac{\mathrm{d}p_n}{p_n}.$$

Using our total derivative of the sectoral price index in equation (E.28) above, we get:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{m=1}^{N} s_{nm} \left[\frac{1}{\sigma - 1} \frac{\mathrm{d}F_i}{F_i} + \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} \right], \tag{E.32}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm} \left[\frac{1}{\sigma - 1} \mathrm{d}\ln F_i + \mathrm{d}\ln \rho_{ni} \right].$$

E.2.7 Wages and Productivity Shocks

We consider small shocks to productivity, holding constant bilateral trade costs and fixed costs:

$$\mathrm{d}\ln\tau_{ni} = 0, \qquad \mathrm{d}\ln F_i = 0, \qquad \forall\, n, i \in N. \tag{E.33}$$

We start with our expression for the log change in wages from equation (E.30) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\begin{array}{c} \mathrm{d}\ln w_n + \left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln F_h - \mathrm{d}\ln F_i\right) \\ +(\sigma-1)\left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln\rho_{nh} - \mathrm{d}\ln\rho_{ni}\right) \end{array} \right),$$

Using the total derivative of the price term (E.24) and our assumptions of constant bilateral trade costs and constant fixed costs (E.33), we can write this expression for the log change in wages as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + (\sigma-1)\left(\sum_{h=1}^{N} s_{nh}\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] \right) \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\sum_{n=1}^{N} t_{in} \left(\sum_{h=1}^{N} s_{nh}\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\left(\sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}s_{nh}\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] \right), \tag{E.34}$$

which can be re-written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\left(\sum_{n=1}^{N} m_{in}\left[\mathrm{d}\ln w_n - \mathrm{d}\ln z_n\right] \right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih}s_{hn} - 1_{n=i},$$

which has the same matrix representation as in equation (15) in the paper:

$$\mathrm{d}\ln\mathbf{w} = \mathbf{T}\,\mathrm{d}\ln\mathbf{w} + \theta\mathbf{M}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right), \tag{E.35}$$

$$\theta = \sigma - 1.$$

E.2.8 Welfare and Productivity Shocks

From equation (E.32), the log change in utility is given by:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni}\left[\frac{1}{\sigma-1}\,\mathrm{d}\ln F_i + \mathrm{d}\ln\rho_{ni}\right],$$

Using the total derivative of the price term (E.24) and our assumptions of constant bilateral trade costs and constant fixed costs (E.33), we can write this log change in utility as:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni}\left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right], \tag{E.36}$$

which has the same matrix representation as in equation (19) in the paper:

$$\mathrm{d}\ln\mathbf{u} = \mathrm{d}\ln\mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right). \tag{E.37}$$

E.2.9 Wages and Fixed Cost Shocks

We next consider small shocks to fixed costs, holding constant bilateral trade costs and productivity:

$$\mathrm{d}\ln\tau_{ni} = 0, \qquad \mathrm{d}\ln z_i = 0, \qquad \forall\, n, i \in N. \tag{E.38}$$

We start with our expression for the log change in wages from equation (E.30) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\begin{array}{c} \mathrm{d}\ln w_n + \left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln F_h - \mathrm{d}\ln F_i\right) \\ +(\sigma-1)\left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln\rho_{nh} - \mathrm{d}\ln\rho_{ni}\right) \end{array} \right),$$

Using the total derivative of the price term (E.24) and our assumptions of constant bilateral trade costs and constant productivity (E.38), we can write this expression for the log change in wages as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\begin{array}{c} \mathrm{d}\ln w_n + \left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln F_h - \mathrm{d}\ln F_i\right) \\ +(\sigma-1)\left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln w_h - \mathrm{d}\ln w_i\right) \end{array} \right),$$

$$\mathrm{d}\ln w_i = \left[\begin{array}{c} \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \sum_{n=1}^{N} t_{in}\left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln F_h - \mathrm{d}\ln F_i\right) \\ +(\sigma-1)\sum_{n=1}^{N} t_{in}\left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln w_h - \mathrm{d}\ln w_i\right) \end{array} \right],$$

$$\mathrm{d}\ln w_i = \left[\begin{array}{c} \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \left(\sum_{n=1}^{N} t_{in}\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln F_h - \mathrm{d}\ln F_i\right) \\ +(\sigma-1)\left(\sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}s_{nh}\,\mathrm{d}\ln w_h - \mathrm{d}\ln w_i\right) \end{array} \right], \tag{E.39}$$

which can be re-written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \sum_{n=1}^{N} m_{in}\,\mathrm{d}\ln F_n + (\sigma-1)\left(\sum_{n=1}^{N} m_{in}\,\mathrm{d}\ln w_n\right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih}s_{hn} - 1_{n=i},$$

which has a similar matrix representation to that for productivity shocks above:

$$\mathrm{d}\ln\mathbf{w} = \mathbf{T}\,\mathrm{d}\ln\mathbf{w} + \theta\mathbf{M}\left(\mathrm{d}\ln\mathbf{w} + \frac{1}{\theta}\,\mathrm{d}\ln\mathbf{F}\right), \tag{E.40}$$

$$\theta = \sigma - 1.$$

E.2.10 Welfare and Fixed Cost Shocks

From equation (E.32), the log change in utility is given by:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni}\left[\frac{1}{\sigma-1}\,\mathrm{d}\ln F_i + \mathrm{d}\ln\rho_{ni}\right],$$

Using the total derivative of the price term (E.24) and our assumptions of constant bilateral trade costs and constant productivity (E.38), we can write this log change in utility as:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni}\left[\frac{1}{\sigma-1}\,\mathrm{d}\ln F_i + \mathrm{d}\ln w_i\right], \tag{E.41}$$

which has a similar matrix representation to that for productivity shocks above:

$$\mathrm{d}\ln\mathbf{u} = \mathrm{d}\ln\mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln\mathbf{w} + \frac{1}{\theta}\,\mathrm{d}\ln\mathbf{F}\right). \tag{E.42}$$

$$\theta = \sigma - 1.$$

F Extensions

In Subsection F.1, we derive the corresponding friend-enemy matrix representations with both productivity and trade cost shocks, as discussed in Section 4.1 of the paper. In Section F.2, we relax one of the ACR macro restrictions to allow for trade imbalance, as discussed in Section 4.2 of the paper. In Section F.3, we relax another of the ACR macro restrictions to consider small deviations from a constant elasticity import demand system, as discussed in Section 4.3 of the paper. In Section F.4, we show that our results generalize to a multi-sector Armington model with a single constant trade elasticity, as discussed in Section 4.4 of the paper. In Section F.5, we show that our results also hold in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). In Section F.6, we further extend the multi-sector specification to introduce input-output linkages following Caliendo and Parro (2015), as discussed in Section 4.5 of the paper. Finally, in Section F.7, we show that our results also hold for economic geography models with factor mobility, including Helpman (1998), Redding and Sturm (2008), Allen and Arkolakis (2014), Ramondo et al. (2016), and Redding (2016), as discussed in Section 4.6 of the paper.

F.1 Trade Cost Reductions

We now show that we obtain similar results incorporating trade cost reductions. We start with our expression for the log change in wages from equation (D.8) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\left(\mathrm{d}\ln w_n + (\sigma-1)\left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni}\right)\right),$$

which can be re-written as follows:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\left(\mathrm{d}\ln w_n + (\sigma-1)\left(\begin{array}{c}\sum_{h=1}^{N} s_{nh}\left[\mathrm{d}\ln w_h + \mathrm{d}\ln\tau_{nh} - \mathrm{d}\ln z_h\right]\\ -\left[\mathrm{d}\ln w_i + \mathrm{d}\ln\tau_{ni} - \mathrm{d}\ln z_i\right]\end{array}\right)\right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\sum_{n=1}^{N} t_{in}\left(\begin{array}{c}\sum_{h=1}^{N} s_{nh}\left[\mathrm{d}\ln w_h + \mathrm{d}\ln\tau_{nh} - \mathrm{d}\ln z_h\right]\\ -\left[\mathrm{d}\ln w_i + \mathrm{d}\ln\tau_{ni} - \mathrm{d}\ln z_i\right]\end{array}\right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\left(\begin{array}{c}\sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}s_{nh}\left[\mathrm{d}\ln w_h + \mathrm{d}\ln\tau_{nh} - \mathrm{d}\ln z_h\right]\\ -\sum_{n=1}^{N} t_{in}\left[\mathrm{d}\ln w_i + \mathrm{d}\ln\tau_{ni} - \mathrm{d}\ln z_i\right]\end{array}\right).$$

We now define inward and outward measures of trade costs as:

$$\mathrm{d}\ln\tau_n^{\mathbf{in}} \equiv \sum_i s_{ni}\,\mathrm{d}\ln\tau_{ni}, \tag{F.1}$$

$$\mathrm{d}\ln\tau_i^{\mathbf{out}} \equiv \sum_n t_{in}\,\mathrm{d}\ln\tau_{ni}. \tag{F.2}$$

Using these definitions of inward and outward trade costs, we can rewrite the above proportional change in wages as follows:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\left(\begin{array}{c}\sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}s_{nh}\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right]\\ +\sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln\tau_n^{\mathbf{in}} - \mathrm{d}\ln\tau_i^{\mathbf{out}}\end{array}\right), \tag{F.3}$$

which can be re-written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\left(\begin{array}{c}\sum_{n=1}^{N} m_{in}\left[\mathrm{d}\ln w_n - \mathrm{d}\ln z_n\right]\\ +\sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln\tau_n^{\mathbf{in}} - \mathrm{d}\ln\tau_i^{\mathbf{out}}\end{array}\right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i},$$

which has the following matrix representation from equation (31) in the paper:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\,\mathrm{d}\ln \mathbf{w} + \theta\left[\mathbf{M}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right) + \mathbf{T}\,\mathrm{d}\ln \tau^{\mathbf{in}} - \mathrm{d}\ln \tau^{\mathbf{out}}\right], \tag{F.4}$$

$$\theta = \sigma - 1.$$

We next consider our expression for the log change in utility from equation (D.10) above:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\,\mathrm{d}\ln p_{nm},$$

which can be re-written as follows:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\left[\mathrm{d}\ln w_m + \mathrm{d}\ln \tau_{nm} - \mathrm{d}\ln z_m\right].$$

Using our definition of inward trade costs from equation (F.1), we can re-write this proportional change in welfare as:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\left(\mathrm{d}\ln w_m - \mathrm{d}\ln z_m\right) - \mathrm{d}\ln \tau_n^{\mathbf{in}},$$

which has the following matrix representation from equation (32) in the paper:

$$\mathrm{d}\ln \mathbf{u} = \mathrm{d}\ln \mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right) + \mathrm{d}\ln \tau^{\mathbf{in}}. \tag{F.5}$$

F.2 Trade Imbalance

In this section of the online appendix, we relax another of the ACR macro restrictions to allow for trade imbalance. In particular, we consider the constant elasticity Armington model from Section 3 of the paper, but allow expenditure to differ from income.

F.2.1 Preferences and Expenditure Shares

We measure the instantaneous welfare of the representative agent as the real value of expenditure:

$$u_n = \frac{w_n \ell_n + \bar{d}_n}{p_n}, \qquad p_n = \left[\sum_{i=1}^{N} p_{ni}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1, \tag{F.6}$$

where $\bar{d}_n$ is the nominal trade deficit. Expenditure shares take the same form as in equation (12) in Section 3 of the paper.

F.2.2 Market Clearing

Market clearing requires that income in each location equals expenditure on goods produced in that location:

$$w_i \ell_i = \sum_{n=1}^{N} s_{ni}\left[w_n \ell_n + \bar{d}_n\right]. \tag{F.7}$$

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

$$\begin{aligned}
\frac{\mathrm{d}w_i}{w_i} w_i \ell_i &= \sum_{n=1}^{N} \frac{\mathrm{d}s_{ni}}{s_{ni}} s_{ni} \left[w_n \ell_n + \bar{d}_n \right] + \sum_{n=1}^{N} s_{ni} \frac{\mathrm{d}w_n}{w_n} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} \bar{d}_n, \\
&= \sum_{n=1}^{N} s_{ni} w_n \ell_n \left(\frac{\mathrm{d}s_{ni}}{s_{ni}} + \frac{\mathrm{d}w_n}{w_n} \right) + \sum_{n=1}^{N} s_{ni} \bar{d}_n \left(\frac{\mathrm{d}s_{ni}}{s_{ni}} + \frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} \right), \\
&= \sum_{n=1}^{N} s_{ni} \left(w_n \ell_n + \bar{d}_n \right) \frac{w_n \ell_n}{\left(w_n \ell_n + \bar{d}_n \right)} \left(\frac{\mathrm{d}s_{ni}}{s_{ni}} + \frac{\mathrm{d}w_n}{w_n} \right) + \sum_{n=1}^{N} s_{ni} \left(w_n \ell_n + \bar{d}_n \right) \frac{\bar{d}_n}{\left(w_n \ell_n + \bar{d}_n \right)} \left(\frac{ds_{ni}}{s_{ni}} + \frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} \right),
\end{aligned}$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} \frac{s_{ni} \left(w_n \ell_n + \bar{d}_n \right)}{w_i \ell_i} \frac{w_n \ell_n}{\left(w_n \ell_n + \bar{d}_n \right)} \left(\frac{\mathrm{d}s_{ni}}{s_{ni}} + \frac{\mathrm{d}w_n}{w_n} \right) + \sum_{n=1}^{N} \frac{s_{ni} \left(w_n \ell_n + \bar{d}_n \right)}{w_i \ell_i} \frac{\bar{d}_n}{\left(w_n \ell_n + \bar{d}_n \right)} \left(\frac{\mathrm{d}s_{ni}}{s_{ni}} + \frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} \right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} t_{in} d_n \left(\frac{\mathrm{d}s_{ni}}{s_{ni}} + \frac{\mathrm{d}w_n}{w_n} \right) + \sum_{n=1}^{N} t_{in} \left(1 - d_n \right) \left(\frac{\mathrm{d}s_{ni}}{s_{ni}} + \frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} \right),$$

where we have defined t_{in} as the share of country i's income from market n:

$$t_{in} \equiv \frac{s_{ni} \left(w_n \ell_n + \bar{d}_n \right)}{w_i \ell_i}$$

and d_n as country n's ratio of income to expenditure:

$$d_n \equiv \frac{w_n \ell_n}{w_n \ell_n + \bar{d}_n}.$$

Using our result for the derivative of expenditure shares above, we can rewrite this as:

$$\begin{aligned}
\frac{\mathrm{d}w_i}{w_i} &= \sum_{n=1}^{N} t_{in} d_n \left(\frac{\mathrm{d}w_n}{w_n} + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}} \right) \right) \\
&\quad + \sum_{n=1}^{N} t_{in} \left(1 - d_n \right) \left(\frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}} \right) \right).
\end{aligned} \tag{F.8}$$

We can re-write this expression as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(d_n \, \mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \, \mathrm{d}\ln p_{nh} - \, \mathrm{d}\ln p_{ni} \right) \right) + \sum_{n=1}^{N} t_{in} \left(1 - d_n \right) \, \mathrm{d}\ln \bar{d}_n.$$

F.2.3 Welfare

Returning to our expression for welfare (F.6), we have:

$$u_n = \frac{w_n \ell_n + \bar{d}_n}{p_n}.$$

Totally differentiating welfare, holding labor endowments constant, we have:

$$\mathrm{d}u_n = \frac{\frac{\mathrm{d}w_n}{w_n} w_n L_n + \frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} \bar{d}_n}{p_n} - \frac{\frac{\mathrm{d}p_n}{p_n} \left[w_n \ell_n + \bar{d}_n \right]}{p_n},$$

$$\begin{aligned}
\frac{\mathrm{d}u_n}{u_n} &= \frac{w_n \ell_n}{w_n \ell_n + \bar{d}_n} \frac{\mathrm{d}w_n}{w_n} + \frac{\bar{d}_n}{w_n \ell_n + \bar{d}_n} \frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} - \frac{\mathrm{d}p_n}{p_n}, \\
&= d_n \frac{\mathrm{d}w_n}{w_n} + \left(1 - d_n \right) \frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} - \frac{dp_n}{p_n}.
\end{aligned}$$

Using the total derivative of the sectoral price index, we get:

$$\frac{\mathrm{d}u_n}{u_n} = d_n \frac{dw_n}{w_n} + (1-d_n)\frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} - \sum_{m=1}^{N} s_{nm}\frac{dp_{nm}}{p_{nm}}, \tag{F.9}$$

which can be re-written as:

$$\mathrm{d}\ln u_n = d_n \,\mathrm{d}\ln w_n + (1-d_n)\,\mathrm{d}\ln \bar{d}_n - \sum_{m=1}^{N} s_{nm} d\ln p_{nm}.$$

F.2.4 Wages and Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

$$\mathrm{d}\ln \tau_{ni} \quad = 0, \quad \forall\, n, i \in N, \tag{F.10}$$

and we assume that trade deficits remain constant in terms of our numeraire of world GDP:

$$\mathrm{d}\ln \bar{d}_n = 0.$$

Under these assumptions, the change in prices is:

$$\mathrm{d}\ln p_{ni} = \mathrm{d}\ln w_i - \mathrm{d}\ln z_i.$$

We start with our expression for the log change in wages in equation (F.8) above. With constant trade deficits ($\mathrm{d}\ln \bar{d}_n = 0$), this expression simplifies to:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\left(d_n \,\mathrm{d}\ln w_n + (\sigma-1)\left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni}\right)\right).$$

Using the total derivative of prices and our assumption of constant bilateral trade costs (F.10), this further simplifies to:

$$\begin{aligned}
\mathrm{d}\ln w_i &= \sum_{n=1}^{N} t_{in}\left(d_n \,\mathrm{d}\ln w_n + (\sigma-1)\left(\sum_{h=1}^{N} s_{nh}\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right]\right)\right), \\
\mathrm{d}\ln w_i &= \sum_{n=1}^{N} d_n t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\left(\sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}s_{nh}\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right]\right). \qquad \text{(F.11)} \\
\mathrm{d}\ln w_i &= \sum_{n=1}^{N} d_n t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\left(\sum_{n=1}^{N} m_{in}\left[\mathrm{d}\ln w_n - \mathrm{d}\ln z_n\right]\right), \\
m_{in} &= \sum_{h=1}^{N} t_{ih}s_{hn} - 1_{n=i},
\end{aligned}$$

which has the following matrix representation:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{TD}\,\mathrm{d}\ln \mathbf{w} + \theta\mathbf{M}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right), \tag{F.12}$$

$$\theta = \sigma - 1$$

where the matrices $\mathbf{T}$ and $\mathbf{M}$ are defined in Section D of this online appendix. The diagonal matrix $\mathbf{D}$ captures trade deficits through the ratio of income to expenditure

$$\mathbf{D} = \underbrace{\begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & d_N \end{pmatrix}}_{N\times N}, \qquad d_n = \frac{w_n \ell_n}{w_n\ell_n + \bar{d}_n}.$$

F.2.5 Welfare and Productivity Shocks

We start with our expression for the log change in welfare in equation (F.9) above. With constant trade deficits ($\mathrm{d}\ln \bar{d}_n = 0$), this simplifies to:

$$\mathrm{d}\ln u_n = d_n \,\mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm} \,\mathrm{d}\ln p_{nm}.$$

Using the total derivative of prices and our assumption of constant bilateral trade costs (F.10), we can write this proportional change in utility as:

$$\mathrm{d}\ln u_n = d_n \,\mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm} \left(\mathrm{d}\ln w_m - d\ln z_m \right), \tag{F.13}$$

which has the following matrix representation:

$$\mathrm{d}\ln \mathbf{u} = \mathbf{D}\,\mathrm{d}\ln \mathbf{w} - \mathbf{S} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right), \tag{F.14}$$

where the matrix $\mathbf{S}$ is defined in Section D of this online appendix and the diagonal matrix $\mathbf{D}$ is defined in the previous subsection of this online appendix.

F.3 Deviations from Constant Elasticity Import Demand

In this section of the online appendix, we consider the generalization of the constant elasticity Armington model in the previous section to incorporate small deviations from a constant elasticity import demand system, as discussed in Section 4.3 of the paper.

F.3.1 Expenditure Shares

We start from the total derivative for expenditure shares in the Armington model with a general homothetic consumption goods price index in equation (C.4) in Section C of this online appendix, as reproduced below:

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = \sum_{h=1}^{N} \left[\theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nkh} \right] \frac{\mathrm{d}p_{nh}}{p_{nh}}, \tag{F.15}$$

where θ_{nih} is the elasticity of expenditure in country n on the good produced by country i with respect to the price of the good produced by country h. In the special case of a constant elasticity import demand, we have:

$$\theta_{nih} = \begin{cases} (s_{nh} - 1)(\sigma - 1) & \text{if } i = h \\ s_{nh}(\sigma - 1) & \text{otherwise} \end{cases}. \tag{F.16}$$

Using this result in equation (F.15), we obtain the total derivative for expenditure shares with a constant elasticity import demand system in equation (D.4) in Section D of this online appendix, as reproduced below:

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}} \right). \tag{F.17}$$

We now allow for deviations from a constant elasticity import demand system by considering the following generalization of the specification in equation (F.16):

$$\theta_{nih} = \begin{cases} (s_{nh} - 1)(\sigma - 1) + o_{nih} & \text{if } i = h \\ s_{nh}(\sigma - 1) + o_{nih} & \text{otherwise} \end{cases}. \tag{F.18}$$

We assume that this generalized demand specification remains homothetic, which implies:

$$
\begin{aligned}
0 &= \sum_{k} x_{nk} \theta_{nkh}, \\
&= \sum_{k=1}^{N} x_{nk} \left((\sigma - 1) s_{nh} + o_{nkh} \right) - (\sigma - 1) x_{nh}, \\
&= \sum_{k=1}^{N} x_{nk} o_{nkh},
\end{aligned}
$$

where x_{ni} denotes country n's expenditure on the goods produced by country i. Therefore, homotheticity implies:

$$
\sum_{k=1}^{N} s_{nk} o_{nkh} = 0. \tag{F.19}
$$

Using our generalized demand specification (F.18) in the total derivative of expenditure shares in equation (F.15), we obtain:

$$
\begin{aligned}
\frac{\mathrm{d}s_{ni}}{s_{ni}} &= \sum_{h=1}^{N} \left[\theta_{nih} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] \frac{\mathrm{d}p_{nh}}{p_{nh}}, \\
&= (\sigma - 1) \left\{ \sum_{h=1}^{N} \left[\left(s_{nh} + \frac{o_{nih}}{\sigma - 1} - 1_{i=h} \right) - \sum_{k=1}^{N} s_{nk} \left(s_{nh} + \frac{o_{nkh}}{\sigma - 1} \right) - s_{nh} \right] \frac{\mathrm{d}p_{nh}}{p_{nh}} \right\}, \\
&= (\sigma - 1) \left\{ \sum_{h=1}^{N} \left[s_{nh} + \frac{o_{nih}}{\sigma - 1} - \sum_{k=1}^{N} s_{nk} \frac{o_{nkh}}{\sigma - 1} \right] \frac{\mathrm{d}p_{nh}}{p_{nh}} - (1 + o_{nii}) \frac{\mathrm{d}p_{ni}}{p_{ni}} \right\}.
\end{aligned}
$$

Using our implication of homotheticity in equation (F.19), this expression simplifies to:

$$
\frac{\mathrm{d}s_{ni}}{s_{ni}} = \left\{ \sum_{h=1}^{N} \left[(\sigma - 1) s_{nh} + o_{nih} \right] \frac{\mathrm{d}p_{nh}}{p_{nh}} - \left[(\sigma - 1) + o_{nii} \right] \frac{\mathrm{d}p_{ni}}{p_{ni}} \right\}. \tag{F.20}
$$

F.3.2 Market Clearing

Market clearing again requires that income in each country equals expenditure on goods produced in that country:

$$
w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \tag{F.21}
$$

Totally differentiating this market clearing condition, we obtain:

$$
\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} \frac{s_{ni} w_n \ell_n}{w_i \ell_i} \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}} \right). \tag{F.22}
$$

Using our result for the derivative of expenditure shares (F.20) in the above equation, we obtain:

$$
\begin{aligned}
\frac{\mathrm{d}w_i}{w_i} &= \sum_{n=1}^{N} \frac{s_{ni} w_n \ell_n}{w_i \ell_i} \left(\frac{\mathrm{d}w_n}{w_n} + \left(\sum_{h=1}^{N} \left[(\sigma - 1) s_{nh} + o_{nih} \right] \frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}} \right) \right), \\
\frac{\mathrm{d}w_i}{w_i} &= \sum_{n=1}^{N} t_{in} \left(\frac{\mathrm{d}w_n}{w_n} + \left(\sum_{h=1}^{N} \left[(\sigma - 1) s_{nh} + o_{nih} \right] \frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}} \right) \right), \\
\mathrm{d} \ln w_i &= \sum_{n=1}^{N} t_{in} \left(\mathrm{d} \ln w_n + \left(\sum_{h=1}^{N} \left[(\sigma - 1) s_{nh} + o_{nih} \right] \mathrm{d} \ln p_{nh} - d \ln p_{ni} \right) \right),
\end{aligned} \tag{F.23}
$$

where we have defined t_{in} as the share of country i's income derived from market n:

$$
t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}.
$$

F.3.3 Welfare

From equation (C.8) in the Armington model with a general homothetic consumption goods price index, we also have the following expression for the total derivative of welfare:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni}\left[\mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right], \tag{F.24}$$

F.3.4 Wages and Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

$$\mathrm{d}\ln \tau_{ni} \quad = 0, \quad \forall\, n, i \in N. \tag{F.25}$$

We start with our expression for the log change in wages in equation (F.23) above. Using the total derivative of prices and our assumption of constant bilateral trade costs (F.25), we can write this expression for the log change in wages as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\left(\mathrm{d}\ln w_n + \left(\sum_{h=1}^{N}\left[(\sigma-1)\, s_{nh} + o_{nih}\right]\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right]\right)\right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \sum_{n=1}^{N} t_{in}\left(\sum_{h=1}^{N}\left[(\sigma-1)\, s_{nh} + o_{nih}\right]\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right]\right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \left(\sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}\left[(\sigma-1)\, s_{nh} + o_{nih}\right]\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right]\right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \left(\begin{array}{c} (\sigma-1)\sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}s_{nh}\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] \\ +\sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}o_{nih}\left(\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right]\right) \end{array}\right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \left(\begin{array}{c} (\sigma-1)\sum_{n=1}^{N} m_{in}\left(\mathrm{d}\ln w_n - \mathrm{d}\ln z_n\right) \\ +\sum_{n=1}^{N} o_{in}\left(\mathrm{d}\ln w_n - \mathrm{d}\ln z_n\right) \end{array}\right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih}s_{hn} - 1_{n=i}, \qquad o_{in} \equiv \sum_{h=1}^{N} t_{ih}o_{hin}$$

which has the following matrix representation in equation (41) in the paper:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\,\mathrm{d}\ln \mathbf{w} + (\theta\mathbf{M} + \mathbf{O})\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right), \tag{F.26}$$

$$\theta = (\sigma - 1).$$

F.3.5 Welfare and Productivity Shocks

Using our assumption of constant bilateral trade costs (F.25) in our expression for the log change in welfare (F.24) above, we obtain:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\left(\mathrm{d}\ln w_m - \mathrm{d}\ln z_m\right), \tag{F.27}$$

which the following matrix representation in equation (42) in the paper:

$$\mathrm{d}\ln \mathbf{u} = \mathrm{d}\ln \mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right). \tag{F.28}$$

F.4 Multiple Sectors

In this section of the online appendix, we show that our friends-and-enemies exposure measures extend naturally to a multi-sector version of the constant elasticity Armington model, as discussed in Section 4.4 of the paper. The world economy consists of many countries indexed by $i, n \in \{1, \ldots, N\}$ and a set of sectors indexed by $k \in \{1, \ldots, K\}$. Each country n has an exogenous supply of labor ℓ_n.

F.4.1 Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

$$u_n = \frac{w_n}{\prod_{k=1}^{K} \left(p_n^k\right)^{\alpha_n^k}}, \qquad \sum_{k=1}^{K} \alpha_n^k = 1. \tag{F.29}$$

Each sector is characterized by constant elasticity of substitution preferences across country varieties:

$$p_n^k = \left[\sum_{i=1}^{N} \left(p_{ni}^k\right)^{-\theta}\right]^{-\frac{1}{\theta}}, \qquad \theta = \sigma - 1, \qquad \sigma > 1. \tag{F.30}$$

F.4.2 Production Technology

Goods are produced with labor under conditions of perfect competition, such that the cost to a consumer in country n of purchasing the variety of country i within sector k is:

$$p_{ni}^k = \frac{\tau_{ni}^k w_i}{z_i^k}, \tag{F.31}$$

where z_i^k captures productivity and iceberg trade costs satisfy $\tau_{ni}^k > 1$ for $n \neq i$ and $\tau_{nn}^k = 1$.

F.4.3 Expenditure Shares

Using the properties of CES demand, country n's share of expenditure on goods produced in country i within sector k is given by:

$$s_{ni}^k = \frac{\left(p_{ni}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta}}. \tag{F.32}$$

Totally differentiating the expenditure share equation (F.32), we get:

$$\mathrm{d}s_{ni}^k = -\frac{\theta \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} \left(p_{ni}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta}} + \sum_{h=1}^{N} \frac{\left(p_{ni}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta}} \frac{\theta \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} \left(p_{nh}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta}},$$

$$\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} = -\theta \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} + \sum_{h=1}^{N} \frac{\theta \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} \left(p_{nh}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta}},$$

$$\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} = -\theta \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} + \sum_{h=1}^{N} s_{nh}^k \theta \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k},$$

$$\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} = \theta \left(\sum_{h=1}^{N} s_{nh}^k \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} - \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k}\right), \tag{F.33}$$

$$\mathrm{d}\ln s_{ni}^k = \theta \left(\sum_{h=1}^{N} s_{nh}^k \,\mathrm{d}\ln p_{nh}^k - \mathrm{d}\ln p_{ni}^k\right),$$

where, from equilibrium prices in equation (F.31), we have:

$$\frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} = \frac{\mathrm{d}\tau_{ni}^k}{\tau_{ni}^k} + \frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i^k}{z_i^k}, \tag{F.34}$$

$$\mathrm{d}\ln p_{ni}^k = \mathrm{d}\ln \tau_{ni}^k + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i^k.$$

F.4.4 Price Indices

Totally differentiating the sectoral price index (F.30), we have:

$$\mathrm{d}p_n^k = \sum_{m=1}^{N} \frac{\mathrm{d}p_{nm}^k}{p_{nm}^k} \frac{\left(p_{nm}^k\right)^{-\theta}}{\sum_{h=1}^{N}\left(p_{nh}^k\right)^{-\theta}} \left[\sum_{m=1}^{N}\left(p_{nm}^k\right)^{-\theta}\right]^{-\frac{1}{\theta}},$$

$$\frac{\mathrm{d}p_n^k}{p_n^k} = \sum_{m=1}^{N} \frac{\mathrm{d}p_{nm}^k}{p_{nm}^k} \frac{\left(p_{nm}^k\right)^{-\theta}}{\sum_{h=1}^{N}\left(p_{nh}^k\right)^{-\theta}},$$

$$\frac{\mathrm{d}p_n^k}{p_n^k} = \sum_{m=1}^{N} s_{nm}^k \frac{\mathrm{d}p_{nm}^k}{p_{nm}^k}, \tag{F.35}$$

$$\mathrm{d}\ln p_n^k = \sum_{m=1}^{N} s_{nm}^k \,\mathrm{d}\ln p_{nm}^k.$$

F.4.5 Market Clearing

Market clearing requires that income in each location equals expenditure on goods produced in that location:

$$w_i \ell_i = \sum_{n=1}^{N}\sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n. \tag{F.36}$$

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{n=1}^{N}\sum_{k=1}^{K} \alpha_n^k \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} s_{ni}^k w_n \ell_n + \sum_{n=1}^{N}\sum_{k=1}^{K} \alpha_n^k s_{ni}^k \frac{\mathrm{d}w_n}{w_n} w_n \ell_n,$$

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{n=1}^{N}\sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k}\right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i} \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k}\right).$$

Using our result for the derivative of expenditure shares in equation (F.33) above, we can rewrite this as:

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i} \left(\frac{\mathrm{d}w_n}{w_n} + \theta\left(\sum_{h=1}^{N} s_{nh}^k \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} - \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k}\right)\right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N}\sum_{k=1}^{K} t_{in}^k \left(\frac{\mathrm{d}w_n}{w_n} + \theta\left(\sum_{h=1}^{N} s_{nh}^k \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} - \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k}\right)\right), \tag{F.37}$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N}\sum_{k=1}^{K} t_{in}^k \left(\mathrm{d}\ln w_n + \theta\left(\sum_{h=1}^{N} s_{nh}^k d\ln p_{nh}^k - d\ln p_{ni}^k\right)\right),$$

where we have defined t_{in}^k as the share of country i's income derived from market n and industry k:

$$t_{in}^k \equiv \frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i}.$$

F.4.6 Utility Again

Returning to our expression for indirect utility in equation (F.29), we have:

$$u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}.$$

Totally differentiating indirect utility, we have:

$$\mathrm{d}u_n = \frac{\frac{\mathrm{d}w_n}{w_n} w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}} - \sum_{k=1}^{K} \frac{\alpha_n^k \frac{\mathrm{d}p_n^k}{p_n^k} w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}},$$

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \frac{\mathrm{d}p_n^k}{p_n^k}.$$

Using our total derivative of the sectoral price index in equation (F.35) above, we get:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \frac{\mathrm{d}p_{nm}^k}{p_{nm}^k}, \tag{F.38}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \,\mathrm{d}\ln p_{nm}^k.$$

F.4.7 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

$$\begin{aligned} &\mathrm{d}\ln z_i^k = \mathrm{d}\ln z_i, && \forall = k \in K,\ i \in N, \\ &\mathrm{d}\ln \tau_{ni}^k = 0, && \forall\, n, i\ \in N. \end{aligned} \tag{F.39}$$

We start with our expression for the log change in wages from equation (F.37) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}^k \,\mathrm{d}\ln p_{nh}^k - \mathrm{d}\ln p_{ni}^k \right) \right).$$

Using the total derivative of prices (F.34) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (F.39), we can write this expression for the log change in wages as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}^k \left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] \right) \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \,\mathrm{d}\ln w_n + \theta \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(\sum_{h=1}^{N} s_{nh}^k \left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \,\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k s_{nh}^k \left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \,\mathrm{d}\ln w_n + \theta \left(\sum_{n=1}^{N} m_{in} \left[\mathrm{d}\ln w_n - \mathrm{d}\ln z_n\right] \right),$$

$$t_{in} = \sum_{k=1}^{K} t_{in}^k,$$

$$m_{in} = \sum_{h=1}^{N} \sum_{k=1}^{K} t_{ih}^k s_{hn}^k - 1_{n=i},$$

which has the following matrix representation in the paper:

$$\mathrm{d}\ln\mathbf{w} = \mathbf{T}\,\mathrm{d}\ln\mathbf{w} + \theta\mathbf{M}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right). \tag{F.40}$$

We again solve for our friend-enemy income exposure measure by matrix inversion. Dividing both sides of equation (F.40) by $\theta + 1$, we have:

$$\frac{1}{\theta+1}\,\mathrm{d}\ln\mathbf{w} = \frac{1}{\theta+1}\mathbf{T}\,\mathrm{d}\ln\mathbf{w} + \frac{\theta}{\theta+1}\mathbf{M}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right),$$

$$\frac{1}{\theta+1}\left(\mathbf{I} - \mathbf{T} - \theta\mathbf{M}\right)\,\mathrm{d}\ln\mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z}.$$

Now using $\mathbf{M} = \mathbf{TS} - \mathbf{I}$, we have:

$$\frac{1}{\theta+1}\left(\mathbf{I} - \mathbf{T} - \theta\mathbf{TS} + \theta\mathbf{I}\right)\,\mathrm{d}\ln\mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

$$\left(\mathbf{I} - \frac{\mathbf{T} + \theta\mathbf{TS}}{\theta+1}\right)\,\mathrm{d}\ln\mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z}.$$

Using our choice of world GDP as numeraire, which implies $\mathbf{Q}\,\mathrm{d}\ln\mathbf{w} = 0$, we have:

$$\left(\mathbf{I} - \frac{\mathbf{T} + \theta\mathbf{TS}}{\theta+1} + \mathbf{Q}\right)\,\mathrm{d}\ln\mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

which can be re-written as:

$$\left(\mathbf{I} - \mathbf{V}\right)\,\mathrm{d}\ln\mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

$$\mathbf{V} \equiv \frac{\mathbf{T} + \theta\mathbf{TS}}{\theta+1} - \mathbf{Q},$$

which yields the following solution of the change in wages in response to a productivity shock:

$$\mathrm{d}\ln\mathbf{w} = -\frac{\theta}{\theta+1}\left(\mathbf{I} - \mathbf{V}\right)^{-1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

which can be re-written as:

$$\mathrm{d}\ln\mathbf{w} = \mathbf{W}\,\mathrm{d}\ln\mathbf{z}, \tag{F.41}$$

where $\mathbf{W}$ is our friend-enemy income exposure measure:

$$\mathbf{W} \equiv -\frac{\theta}{\theta+1}\left(\mathbf{I} - \mathbf{V}\right)^{-1}\mathbf{M}. \tag{F.42}$$

F.4.8 Welfare and Common Productivity Shocks

We start with our expression for the log change in utility in equation (F.38) above:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{k=1}^{K}\alpha_n^k\sum_{m=1}^{N}s_{nm}^k\,\mathrm{d}\ln p_{nm}^k,$$

or equivalently:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N}\sum_{k=1}^{K}\alpha_n^k s_{nm}^k\,\mathrm{d}\ln p_{nm}^k.$$

Using the total derivative of prices (F.34) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (F.39), we can write this change in log utility as:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N}\sum_{k=1}^{K}\alpha_n^k s_{nm}^k\left(\mathrm{d}\ln w_m - \mathrm{d}\ln z_m\right), \tag{F.43}$$

which has the following matrix representation in the paper:

$$\mathrm{d}\ln\mathbf{u} = \mathrm{d}\ln\mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right). \tag{F.44}$$

We can re-write the above relationship as:

$$\mathrm{d}\ln\mathbf{u} = (\mathbf{I} - \mathbf{S})\ \mathrm{d}\ln\mathbf{w} + \mathbf{S}\,\mathrm{d}\ln\mathbf{z},$$

which, using our solution for $\mathrm{d}\ln\mathbf{w}$ from equation (F.41), can be further re-written as:

$$\mathrm{d}\ln\mathbf{u} = (\mathbf{I} - \mathbf{S})\,\mathbf{W}\,\mathrm{d}\ln\mathbf{z} + \mathbf{S}\,\mathrm{d}\ln\mathbf{z},$$

$$\mathrm{d}\ln\mathbf{u} = \left[(\mathbf{I} - \mathbf{S})\,\mathbf{W} + \mathbf{S}\right]\ \mathrm{d}\ln\mathbf{z},$$

$$\mathrm{d}\ln\mathbf{u} = \mathbf{U}\,\mathrm{d}\ln\mathbf{z}, \tag{F.45}$$

where $\mathbf{U}$ is our friend-enemy welfare exposure measure:

$$\mathbf{U} \equiv \left[(\mathbf{I} - \mathbf{S})\,\mathbf{W} + \mathbf{S}\right]. \tag{F.46}$$

F.4.9 Industry-Level Sales Exposure

In this multi-sector model, our approach also yields bilateral friend-enemy measures of income exposure to global productivity shocks for each sector. Labor income in each sector and country equals value-added, which in turn equals expenditure on goods produced in that sector and country:

$$w_i \ell_i^k = y_i^k = \sum_{n=1}^{N} \alpha_n^k s_{ni}^k w_n \ell_n.$$

Totally differentiating this industry market clearing condition, we have:

$$\frac{\mathrm{d}y_i^k}{y_i^k} w_i \ell_i = \sum_{n=1}^{N} \alpha_n^k \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} a_{ni}^k w_n \ell_n + \sum_{n=1}^{N} \alpha_n^k s_{ni}^k \frac{\mathrm{d}w_n}{w_n} w_n \ell_n,$$

$$\frac{\mathrm{d}y_i^k}{y_i^k} w_i \ell_i = \sum_{n=1}^{N} \alpha_n^k s_{ni}^k w_n \ell_n \left(\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} + \frac{\mathrm{d}w_n}{w_n} \right),$$

$$\frac{\mathrm{d}y_i^k}{y_i^k} = \sum_{n=1}^{N} t_{in}^k \left(\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} + \frac{\mathrm{d}w_n}{w_n} \right),$$

$$\frac{\mathrm{d}y_i^k}{y_i^k} = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} \right).$$

Using the total derivative of expenditure shares in equation (F.33), we can rewrite this as:

$$\frac{\mathrm{d}y_i^k}{y_i^k} = \sum_{n=1}^{N} t_{in}^k \left(\frac{\mathrm{d}w_n}{w_n} + \theta \left(\sum_{h=1}^{N} s_{nh}^k \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} - \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} \right) \right),$$

which can be re-written as:

$$\mathrm{d}\ln y_i^k = \sum_{n=1}^{N} t_{in}^k \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}^k \,\mathrm{d}\ln p_{nh}^k - \mathrm{d}\ln p_{ni}^k \right) \right).$$

Using the total derivative of prices (F.34) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (F.39), we can re-write this change in sector value-added as:

$$\mathrm{d}\ln y_i^k = \sum_{n=1}^{N} t_{in}^k \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}^k \left(\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right) - \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right) \right) \right),$$

which has the following matrix representation:

$$\mathrm{d}\ln \mathbf{Y^k} = \mathbf{T^k}\,\mathrm{d}\ln \mathbf{w} + \theta \mathbf{M^k} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right). \tag{F.47}$$

Using our solution for changes in wages as a function of productivity shocks in equation (F.40) above, we have:

$$\mathrm{d}\ln \mathbf{Y^k} = \mathbf{T^k}\mathbf{W}\,\mathrm{d}\ln \mathbf{z} + \theta \mathbf{M^k} \left(\mathbf{W} - \mathbf{I} \right) \mathrm{d}\ln \mathbf{z},$$

$$\mathrm{d}\ln \mathbf{Y^k} = \left[\mathbf{T^k}\mathbf{W} + \theta \mathbf{M^k} \left(\mathbf{W} - \mathbf{I} \right) \right] \mathrm{d}\ln \mathbf{z},$$

which can be re-written as:

$$\mathrm{d}\ln \mathbf{Y^k} = \mathbf{W^k}\,\mathrm{d}\ln \mathbf{z}, \tag{F.48}$$

$$\mathbf{W^k} \equiv \mathbf{T^k}\mathbf{W} + \theta \mathbf{M^k} \left(\mathbf{W} - \mathbf{I} \right), \tag{F.49}$$

which corresponds to our friends-and-enemies measure of sector value-added exposure to productivity shocks.

We now show that our aggregate friends-and-enemies measure of income exposure ($\mathbf{W}$) in equation (F.42) is a weighted average of our industry friends-and-enemies measures of sector value-added exposure in equation (F.49). Note that aggregate income equals aggregate value-added:

$$w_i \ell_i = \sum_{k=1}^{K} y_i^k.$$

Totally differentiating, holding endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{k=1}^{K} \frac{\mathrm{d}y_i^k}{y_i^k} y_i^k.$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{k=1}^{K} \frac{y_i^k}{w_i \ell_i} \frac{\mathrm{d}y_i^k}{y_i^k}.$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{k=1}^{K} r_i^k \frac{\mathrm{d}y_i^k}{y_i^k},$$

$$\mathrm{d}\ln w_i = \sum_{k=1}^{K} r_i^k\,\mathrm{d}\ln y_i^k, \tag{F.50}$$

where $r_i^k \equiv \frac{y_i^k}{w_i \ell_i}$ is the value-added share of industry k. Together, equations (F.41), (F.48) and (F.50) imply:

$$\mathbf{W}_i\,\mathrm{d}\ln \mathbf{z} = \sum_{k=1}^{K} r_i^k \mathbf{W}_i^k\,\mathrm{d}\ln \mathbf{z},$$

where $\mathbf{W}_i$ is the income exposure vector for country i with respect to productivity shocks in its trade partners and $\mathbf{W}_i^k$ is the sector value-added exposure vector for country i and sector k with respect to productivity shocks in those trade partners. It follows that our aggregate friends-and-enemies measure of income exposure ($\mathbf{W}_i$) is a weighted average of our industry friends-and-enemies measures of sector income exposure ($\mathbf{W}_i^k$):

$$\mathbf{W}_i = \sum_k r_i^k \mathbf{W}_i^k.$$

Therefore, we can decompose our aggregate income exposure measure into the contributions of the income exposure measures of particular industries, and how much of that income exposure of particular industries is explained by various terms (market-size, cross-substitution etc within the industry).

F.5 Multi-Sector Isomorphism

For the convenience of exposition, we focus in Section 3 of the paper and the previous section of this online appendix on a multi-sector version of the constant elasticity Armington model. In this section, we show that the same results hold in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). The world economy consists of a set of countries indexed by $i, n \in \{1, \ldots, N\}$ and a set of sectors indexed by $k \in \{1, \ldots, K\}$. Each country n has an exogenous supply of labor ℓ_n.

F.5.1 Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

$$u_n = \frac{w_n}{\prod_{k=1}^K \left(p_n^k\right)^{\alpha_n^k}}, \qquad \sum_{k=1}^K \alpha_n^k = 1. \tag{F.51}$$

Each sector contains a fixed continuum of goods that enter the sectoral price index according to the following CES functional form:

$$p_n^k = \left[\int_0^1 p_n^k(\vartheta)^{1-\sigma^k} d\vartheta\right]^{\frac{1}{1-\sigma^k}}, \qquad \sigma^k > 1. \tag{F.52}$$

F.5.2 Production Technology

Goods are produced with labor and can be traded subject to iceberg variable trade costs, such that the cost to a consumer in country n of purchasing a good ϑ from country i is:

$$p_{ni}^k(\vartheta) = \frac{\tau_{ni}^k w_i}{z_i^k a_i^k(\vartheta)}, \tag{F.53}$$

where z_i^k captures determinants of productivity that are common across all goods within a country i and sector k and $a_i^k(\vartheta)$ captures idiosyncratic determinants of productivity for each good within that country and sector. Iceberg trade costs satisfy $\tau_{ni}^k > 1$ for $n \neq i$ and $\tau_{nn}^k = 1$. Productivity for each good ϑ in each sector k and each country i is drawn independently from the following Fréchet distribution:

$$F_i^k(a) = \exp\left(-a^{-\theta}\right), \qquad \theta > 1, \tag{F.54}$$

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to z_i^k.

F.5.3 Expenditure Shares

Using the properties of this Fréchet distribution, country n's share of expenditure on goods produced in country i within sector k is given by:

$$s_{ni}^k = \frac{\left(\tau_{ni}^k w_i / z_i^k\right)^{-\theta}}{\sum_{m=1}^N \left(\tau_{nm}^k w_m / z_m^k\right)^{-\theta}} = \frac{\left(\rho_{ni}^k\right)^{-\theta}}{\sum_{m=1}^N \left(\rho_{nm}^k\right)^{-\theta}}, \tag{F.55}$$

where we have defined the following price term:

$$\rho_{ni}^k \equiv \frac{\tau_{ni}^k w_i}{z_i^k}. \tag{F.56}$$

Totally differentiating the expenditure share equation (F.55), we get:

$$\begin{aligned}
\mathrm{d}s_{ni}^k &= -\frac{\theta \frac{\mathrm{d}\rho_{ni}^k}{\rho_{ni}^k}\left(\rho_{ni}^k\right)^{-\theta}}{\sum_{m=1}^N \left(\rho_{nm}^k\right)^{-\theta}} + \sum_{h=1}^N \frac{\left(\rho_{ni}^k\right)^{-\theta}}{\sum_{m=1}^N \left(\rho_{nm}^k\right)^{-\theta}} \frac{\theta \frac{\mathrm{d}\rho_{nh}^k}{\rho_{nh}^k}\left(\rho_{nh}^k\right)^{-\theta}}{\sum_{m=1}^N \left(\rho_{nm}^k\right)^{-\theta}}, \\
\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} &= -\theta \frac{\mathrm{d}\rho_{ni}^k}{\rho_{ni}^k} + \sum_{h=1}^N \frac{\theta \frac{\mathrm{d}\rho_{nh}^k}{\rho_{nh}^k}\left(\rho_{nh}^k\right)^{-\theta}}{\sum_{m=1}^N \left(\rho_{nm}^k\right)^{-\theta}}, \\
\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} &= -\theta \frac{\mathrm{d}\rho_{ni}^k}{\rho_{ni}^k} + \sum_{h=1}^N s_{nh}^k \theta \frac{\mathrm{d}\rho_{nh}^k}{\rho_{nh}^k}, \\
\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} &= \theta\left(\sum_{h=1}^N s_{nh}^k \frac{\mathrm{d}\rho_{nh}^k}{\rho_{nh}^k} - \frac{\mathrm{d}\rho_{ni}^k}{\rho_{ni}^k}\right), \qquad \text{(F.57)} \\
\mathrm{d}\ln s_{ni}^k &= \theta\left(\sum_{h=1}^N s_{nh}^k \,\mathrm{d}\ln \rho_{nh}^k - \mathrm{d}\ln \rho_{ni}^k\right),
\end{aligned}$$

where from the definition of ρ_{ni}^k above we have:

$$\begin{aligned}
\frac{\mathrm{d}\rho_{ni}^k}{\rho_{ni}^k} &= \frac{\mathrm{d}\tau_{ni}^k}{\tau_{ni}^k} + \frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i^k}{z_i^k}, \qquad \text{(F.58)} \\
\mathrm{d}\ln \rho_{ni}^k &= \mathrm{d}\ln \tau_{ni}^k + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i^k.
\end{aligned}$$

F.5.4 Price Indices

Using the properties of the Fréchet distribution (F.54), the sectoral price index is given by:

$$p_n^k = \gamma^k \left[\sum_{m=1}^N \left(\rho_{nm}^k\right)^{-\theta}\right]^{-\frac{1}{\theta}}, \tag{F.59}$$

where

$$\gamma^k \equiv \left[\Gamma\left(\frac{\theta + 1 - \sigma^k}{\theta}\right)\right]^{\frac{1}{1-\sigma^k}},$$

and $\Gamma(\cdot)$ denotes the Gamma function. Totally differentiating this sectoral price index (F.59), we have:

$$\begin{aligned}
\mathrm{d}p_n^k &= \sum_{m=1}^N \gamma^k \frac{\mathrm{d}\rho_{nm}^k}{\rho_{nm}^k} \frac{\left(\rho_{nm}^k\right)^{-\theta}}{\sum_{h=1}^N \left(\rho_{nh}^k\right)^{-\theta}} \left[\sum_{m=1}^N \left(\rho_{nm}^k\right)^{-\theta}\right]^{-\frac{1}{\theta}}, \\
\frac{\mathrm{d}p_n^k}{p_n^k} &= \sum_{m=1}^N \frac{\mathrm{d}\rho_{nm}^k}{\rho_{nm}^k} \frac{\left(\rho_{nm}^k\right)^{-\theta}}{\sum_{h=1}^N \left(\rho_{nh}^k\right)^{-\theta}}, \\
\frac{\mathrm{d}p_n^k}{p_n^k} &= \sum_{m=1}^N s_{nm}^k \frac{\mathrm{d}\rho_{nm}^k}{\rho_{nm}^k}, \qquad \text{(F.60)} \\
\mathrm{d}\ln p_n^k &= \sum_{m=1}^N s_{nm}^k \,\mathrm{d}\ln \rho_{nm}^k.
\end{aligned}$$

F.5.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

$$w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n. \tag{F.61}$$

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} s_{ni}^k w_n \ell_n + \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k \frac{\mathrm{d}w_n}{w_n} w_n \ell_n,$$

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} \right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i} \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} \right).$$

Using our result for the derivative of expenditure shares in equation (F.57) above, we can rewrite this as:

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i} \left(\frac{\mathrm{d}w_n}{w_n} + \theta \left(\sum_{h=1}^{N} s_{nh}^k \frac{\mathrm{d}\rho_{nh}^k}{\rho_{nh}^k} - \frac{\mathrm{d}\rho_{ni}^k}{\rho_{ni}^k} \right) \right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(\frac{\mathrm{d}w_n}{w_n} + \theta \left(\sum_{h \in N} s_{nh}^k \frac{\mathrm{d}\rho_{nh}^k}{\rho_{nh}^k} - \frac{\mathrm{d}\rho_{ni}^k}{\rho_{ni}^k} \right) \right), \tag{F.62}$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}^k \,\mathrm{d}\ln \rho_{nh}^k - \mathrm{d}\ln \rho_{ni}^k \right) \right),$$

where we have defined t_{in}^k as country n's expenditure on country i in industry k as a share of country i's income:

$$t_{in}^k \equiv \frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i}.$$

F.5.6 Utility Again

Returning to our expression for indirect utility in equation (F.51), we have:

$$u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}.$$

Totally differentiating indirect utility, we have:

$$\mathrm{d}u_n = \frac{\frac{\mathrm{d}w_n}{w_n} w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}} - \sum_{k=1}^{K} \frac{\alpha_n^k \frac{\mathrm{d}p_n^k}{p_n^k} w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}},$$

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \frac{\mathrm{d}p_n^k}{p_n^k}.$$

Using our total derivative of the sectoral price index in equation (F.60) above, we get:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \frac{\mathrm{d}\rho_{nm}^k}{\rho_{nm}^k}, \tag{F.63}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \,\mathrm{d}\ln \rho_{nm}^k.$$

F.5.7 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

$$\begin{aligned} d\ln z_i^k &= d\ln z_i, \quad && \forall\, k \in K,\ i \in N, \\ d\ln \tau_{ni}^k &= 0, && \forall\, n, i\ \in N. \end{aligned} \tag{F.64}$$

We start with our expression for the log change in wages from equation (F.62) above:

$$d\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(d\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}^k \, d\ln \rho_{nh}^k - d\ln \rho_{ni}^k \right) \right).$$

Using the total derivative of prices (F.58) and our assumption of constant bilateral trade costs (F.64), we can write this log change in wages as:

$$d\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(d\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}^k \left(d\ln w_h - d\ln z_h \right) - \left(d\ln w_i - d\ln z_i \right) \right) \right),$$

$$d\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \, d\ln w_n + \theta \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(\sum_{h=1}^{N} s_{nh}^k \left(d\ln w_h - d\ln z_h \right) - \left(d\ln w_i - d\ln z_i \right) \right),$$

$$d\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \, d\ln w_n + \theta \left(\sum_{h=1}^{N} \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k s_{nh}^k \left(d\ln w_h - d\ln z_h \right) - \left(d\ln w_i - d\ln z_i \right) \right), \tag{F.65}$$

which can be re-written as:

$$d\ln w_i = \sum_{n=1}^{N} t_{in} \, d\ln w_n + \theta \left(\sum_{n=1}^{N} m_{in} \left[d\ln w_n - d\ln z_n \right] \right),$$

$$t_{in} = \sum_{k=1}^{K} t_{in}^k,$$

$$m_{in} = \sum_{h=1}^{N} \sum_{k=1}^{K} t_{ih}^k s_{hn}^k - 1_{n=i},$$

which has the same matrix representation as in equation (46) in the paper:

$$d\ln \mathbf{w} = \mathbf{T} \, d\ln \mathbf{w} + \theta \mathbf{M} \left(d\ln \mathbf{w} - d\ln \mathbf{z} \right). \tag{F.66}$$

F.5.8 Utility and Common Productivity Shocks

We start with our expression for the log change in utility in equation (F.63) above:

$$d\ln u_n = d\ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \, d\ln \rho_{nm}^k,$$

or equivalently:

$$d\ln u_n = d\ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{nm}^k \, d\ln \rho_{nm}^k.$$

Using the total derivative of prices (F.58) and our assumption of constant bilateral trade costs (F.64), we can write this change in utility as:

$$d\ln u_n = d\ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{nm}^k \left(d\ln w_m - d\ln z_m \right),$$

which has the same matrix representation as in equation (47) in the paper:

$$d\ln \mathbf{u} = d\ln \mathbf{w} - \mathbf{S} \left(d\ln \mathbf{w} - d\ln \mathbf{z} \right). \tag{F.67}$$

F.6 Multiple Sectors and Input-Output Linkages

In this section of the online appendix, we report the derivations for a version of the model with multiple sectors and input-output linkages following Caliendo and Parro (2015), henceforth CP. In particular, we consider a generalization of the multi-sector Armington model in Section F.4 of this online appendix to incorporate input-output linkages, as discussed in Section 4.5 of the paper. The world economy consists of a set of countries indexed by $i, n \in \{1, \ldots, N\}$ and a set of sectors referenced by $k \in \{1, \ldots, K\}$. Each country n has an exogenous supply of labor ℓ_n.

F.6.1 Notations

We use i, n, o, r to index for countries and j, k, l for industries. We refer to the varieties in industry k produced in country i as "goods ik". We use subscripts to denote countries and superscripts to denote industries. Let $I_{(NK)}$ denote the identity matrix with dimension $NK \times NK$.

F.6.2 Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

$$u_n = \frac{w_n}{\prod_{k=1}^{K} \left(p_n^k\right)^{\alpha_n^k}}, \qquad \sum_{k=1}^{K} \alpha_n^k = 1. \tag{F.68}$$

Each sector is characterized by constant elasticity of substitution preferences across country varieties:

$$p_n^k = \left[\sum_{i=1}^{N} \left(p_{ni}^k\right)^{-\theta}\right]^{-\frac{1}{\theta}}, \qquad \theta = \sigma - 1, \qquad \sigma^k > 1. \tag{F.69}$$

F.6.3 Production Technology

Goods are produced with labor and can be traded subject to iceberg trade costs, such that the cost to a consumer in country n of purchasing country i's variety within sector k is:

$$p_{ni}^k(\omega) = \tau_{ni}^k c_i^k, \qquad c_i^k = \left(\frac{w_i}{z_i^k}\right)^{\gamma_i^k} \prod_{j=1}^{K} \left(p_i^j\right)^{\gamma_i^{k,j}}, \qquad \sum_{k=1}^{K} \gamma_i^{k,j} = 1 - \gamma_i^k, \tag{F.70}$$

where c_i^k denotes the unit cost function within that country and sector; γ_i^k is the share of labor in production costs; $\gamma_i^{k,j}$ is the share of materials from sector j used in sector k; z_i^k captures determinants of productivity that are common across all goods within a country i and sector k; and it proves convenient to define this common component of productivity in value-added terms (such that it augments labor). Iceberg variable trade costs satisfy $\tau_{ni}^k > 1$ for $n \neq i$ and $\tau_{nn}^k = 1$.

F.6.4 Expenditure Shares

Using the properties of CES demand, country n's share of expenditure on goods produced in country i within sector k is given by:

$$s_{ni}^k = \frac{\left(p_{ni}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta}}. \tag{F.71}$$

Totally differentiating this expenditure share equation, we get:

$$\mathrm{d}s_{ni}^k = -\frac{\theta \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} \left(p_{ni}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta}} + \sum_{h=1}^{N} \frac{\left(p_{ni}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta}} \frac{\theta \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} \left(p_{nh}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta}},$$

$$\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} = -\theta \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} + \sum_{h=1}^{N} \frac{\theta \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} \left(p_{nh}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta}},$$

$$\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} = -\theta \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} + \sum_{h=1}^{N} s_{nh}^k \theta \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k},$$

$$\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} = \theta \left(\sum_{h=1}^{N} s_{nh}^k \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} - \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} \right).$$

so that

$$\mathrm{d}\ln s_{ni}^k = \theta \left(\sum_{h=1}^{N} s_{nh}^k \, \mathrm{d}\ln p_{nh}^k - \mathrm{d}\ln p_{ni}^k \right), \tag{F.72}$$

where, from equilibrium prices in equation (F.70), we have:

$$\frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} = \frac{\mathrm{d}\tau_{ni}^k}{\tau_{ni}^k} + \gamma_i^k \left(\frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i^k}{z_i^k} \right) + \sum_{j=1}^{K} \gamma_i^{k,j} \frac{\mathrm{d}p_i^j}{p_i^j}, \tag{F.73}$$

$$\mathrm{d}\ln p_{ni}^k = \mathrm{d}\ln \tau_{ni}^k + \gamma_i^k \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i^k \right) + \sum_{j=1}^{K} \gamma_i^{k,j} \, \mathrm{d}\ln p_i^j.$$

F.6.5 Price Indices

Totally differentiating the sectoral price index (F.69), we have:

$$\mathrm{d}p_n^k = \sum_{m=1}^{N} \gamma^k \frac{\mathrm{d}p_{nm}^k}{p_{nm}^k} \frac{\left(p_{nm}^k\right)^{-\theta}}{\sum_{h=1}^{N} \left(p_{nh}^k\right)^{-\theta}} \left[\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta} \right]^{-\frac{1}{\theta}},$$

$$\frac{\mathrm{d}p_n^k}{p_n^k} = \sum_{m=1}^{N} \frac{\mathrm{d}p_{nm}^k}{p_{nm}^k} \frac{\left(p_{nm}^k\right)^{-\theta}}{\sum_{h=1}^{N} \left(p_{nh}^k\right)^{-\theta}},$$

$$\frac{\mathrm{d}p_n^k}{p_n^k} = \sum_{m=1}^{N} s_{nm}^k \frac{\mathrm{d}p_{nm}^k}{p_{nm}^k}, \tag{F.74}$$

$$\mathrm{d}\ln p_n^k = \sum_{m=1}^{N} s_{nm}^k \, \mathrm{d}\ln p_{nm}^k.$$

F.6.6 Labor Market Clearing

The labor market clearing condition is:

$$w_n \ell_n = \sum_{j=1}^{K} \gamma_n^j y_n^j. \tag{F.75}$$

where y_n^j is total sales by country n's industry j. Totally differentiating this labor market clearing condition, holding endowments constant, we have:

$$\frac{\mathrm{d}w_n}{w_n} w_n \ell_n = \sum_{j=1}^{K} \gamma_n^j \frac{\mathrm{d}y_n^j}{y_n^j} y_n^j,$$

$$\frac{\mathrm{d}w_n}{w_n} = \sum_{j=1}^{K} \left(\frac{\gamma_n^j y_n^j}{w_n L_n} \right) \frac{\mathrm{d}y_n^j}{y_n^j},$$

$$\frac{\mathrm{d}w_n}{w_n} = \sum_{j=1}^{K} \xi_n^j \frac{\mathrm{d}y_n^j}{y_n^j}, \tag{F.76}$$

where ξ_n^j is the share of sector j in country n's total income:

$$\xi_n^j \equiv \frac{\gamma_n^j y_n^j}{w_n L_n}.$$

F.6.7 Goods Market Clearing

Goods market clearing requires that income in each location and sector equals expenditure on goods produced in that location and sector:

$$y_i^k = \sum_{n=1}^{N} s_{ni}^k x_n^k, \tag{F.77}$$

where expenditure in location n in sector k is:

$$x_n^k = \alpha_n^k w_n \ell_n + \sum_{j=1}^{K} \gamma_n^{j,k} y_n^j, \tag{F.78}$$

and recall that $\gamma_n^{j,k}$ is the share of materials from sector k used in sector j. Combining these two relationships and the labor market clearing (F.75), we obtain the following market clearing condition:

$$\begin{aligned}
y_i^k &= \sum_{n=1}^{N} s_{ni}^k \left[\alpha_n^k w_n \ell_n + \sum_{j=1}^{K} \gamma_n^{k,j} y_n^j \right], \\
&= \sum_{n=1}^{N} s_{ni}^k \left[\alpha_n^k \sum_{j=1}^{K} \gamma_n^j y_n^j + \sum_{j=1}^{K} \gamma_n^{k,j} y_n^j \right], \\
&= \sum_{n=1}^{N} \sum_{j=1}^{K} s_{ni}^k \left[\alpha_n^k \gamma_n^j + \gamma_n^{k,j} \right] y_n^j.
\end{aligned}$$

Totally differentiating this market clearing condition, we have:

$$\begin{aligned}
\frac{\mathrm{d}y_i^k}{y_i^k} y_i^k &= \sum_{n=1}^{N} s_{ni}^k \left[\alpha_n^k w_n \ell_n + \sum_{j=1}^{K} \gamma_n^{k,j} y_n^j \right], \\
&= \sum_{n=1}^{N} s_{ni}^k \left[\alpha_n^k \sum_{j=1}^{K} \gamma_n^j y_n^j + \sum_{j=1}^{K} \gamma_n^{k,j} y_n^j \right], \\
&= \sum_{n=1}^{N} \sum_{j=1}^{K} s_{ni}^k \left[\alpha_n^k \gamma_n^j + \gamma_n^{k,j} \right] y_n^j.
\end{aligned}$$

$$\frac{\mathrm{d}y_i^k}{y_i^k} y_i^k = \sum_{n=1}^{N} \sum_{j=1}^{K} \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} s_{ni}^k \left[\alpha_n^k \gamma_n^j + \gamma_n^{k,j} \right] y_n^j + \sum_{n=1}^{N} \sum_{j=1}^{K} s_{ni}^k \left[\alpha_n^k \gamma_n^j + \gamma_n^{k,j} \right] \frac{\mathrm{d}y_n^j}{y_n^j} y_n^j.$$

Let ϑ_{in}^k denote the fraction of ik's revenue derived from selling to consumers in country n; $\mathbf{\Theta}$ denote an $NK \times NK$ matrix with entries Θ_{in}^{kj} capturing the fraction of ik's revenue derived from selling to producers in country n industry j; and $\mathbf{\Delta}$ denote the Leontief-inverse of Θ, such that $\mathbf{\Delta} \equiv \left(\mathbf{I}_{(NK)} - \mathbf{\Theta}\right)^{-1}$, with the (ik, nj)-th entry, Δ_{in}^{kj}, capturing the network-adjusted fraction of ik's revenue derived from market nj, either directly or indirectly through customers of customers, ad infinitum. The above market clearing can be re-written as

$$\begin{aligned}
\mathrm{d}\ln y_i^k &= \sum_{n=1}^{N} \mathrm{d}\ln s_{ni}^k \frac{s_{ni}^k \alpha_n^k \sum_{j=1}^{K} \gamma_n^j y_n^j}{y_i^k} + \sum_{n=1}^{N} \sum_{j=1}^{K} \mathrm{d}\ln s_{ni}^k \frac{s_{ni}^k \gamma_n^{k,j}}{y_i^k} y_n^j \\
&+ \sum_{n=1}^{N} \frac{s_{ni}^k \alpha_n^k w_n \ell_n}{y_i^k} \sum_{j=1}^{K} \frac{\gamma_n^j y_n^j}{w_n \ell_n} \mathrm{d}\ln y_n^j + \sum_{n=1}^{N} \sum_{j=1}^{K} \frac{s_{ni}^k \gamma_n^{k,j}}{y_i^k} y_n^j \,\mathrm{d}\ln y_n^j, \\
&= \sum_{n=1}^{N} \vartheta_{in}^k \,\mathrm{d}\ln w_n + \sum_{n=1}^{N} \left(\vartheta_{in}^k + \sum_{j=1}^{K} \Theta_{in}^{kj} \right) \mathrm{d}\ln s_{ni}^k + \sum_{n=1}^{N} \sum_{j=1}^{K} \Theta_{in}^{kj} \,\mathrm{d}\ln y_n^j,
\end{aligned}$$

where in the second equality we used equations (F.75) and (F.76). Subtracting the latest term on the right hand side from both sides of the equations and taking the Leontief-inverse of Θ_{nm}^{ik}, we obtain:

$$\mathrm{d}\ln y_i^k = \sum_{o=1}^{N}\sum_{l=1}^{K}\Delta_{io}^{kl}\left[\sum_{n=1}^{N}\vartheta_{on}^{l}\,\mathrm{d}\ln w_n + \sum_{n=1}^{N}\left(\vartheta_{on}^{l} + \sum_{j=1}^{K}\Theta_{on}^{lj}\right)\mathrm{d}\ln s_{no}^{l}\right]. \tag{F.79}$$

Combining this result with (F.76), we get

$$\mathrm{d}\ln w_i = \sum_{k}\xi_i^k\sum_{o=1}^{N}\sum_{l=1}^{K}\Delta_{io}^{kl}\left[\sum_{n=1}^{N}\vartheta_{on}^{l}\,\mathrm{d}\ln w_n + \sum_{n=1}^{N}\left(\vartheta_{on}^{l} + \sum_{j=1}^{K}\Theta_{on}^{lj}\right)\mathrm{d}\ln s_{no}^{l}\right]. \tag{F.80}$$

F.6.8 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

$$\begin{aligned}\mathrm{d}\ln z_i^k &= \mathrm{d}\ln z_i, \quad \forall k\in K,\ i\in N,\\ \mathrm{d}\ln\tau_{ni}^k &= 0, \qquad\quad \forall\, n,i\in N.\end{aligned}$$

We start with our expression for the change in prices above:

$$\begin{aligned}\mathrm{d}\ln p_{ni}^k &= \mathrm{d}\ln\tau_{ni}^k + \gamma_i^k\left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i^k\right) + \sum_{j=1}^{K}\gamma_i^{k,j}\,\mathrm{d}\ln p_i^j,\\ &= \gamma_i^k\left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right) + \sum_{j=1}^{K}\gamma_i^{k,j}\,\mathrm{d}\ln p_i^j\end{aligned}$$

Using our result for the total derivative of price indices (F.74), we can rewrite this expression for the change in prices as:

$$\mathrm{d}\ln p_{ni}^k = \gamma_i^k\left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right) + \sum_{j=1}^{K}\gamma_i^{k,j}\sum_{m=1}^{N}s_{im}^{j}\,\mathrm{d}\ln p_{im}^j.$$

We use $\Sigma_{im}^{j,k} = \gamma_i^{k,j}s_{im}^j$ to denote expenditure in country i and sector k on the goods produced by country m and sector j as a share of revenue in country i and sector k. Using this notation, we can rewrite the above expression for the change in prices as:

$$\mathrm{d}\ln p_{ni}^k = \gamma_i^k\left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right) + \sum_{j=1}^{K}\sum_{m=1}^{N}\Sigma_{im}^{j,k}\,\mathrm{d}\ln p_{im}^j. \tag{F.81}$$

Let $\mathbf{\Sigma}$ denote the $NK\times NK$ matrix with entries $\Sigma_{im}^{j,k}$ capturing the input cost share (relative to revenue) on goods km by producer ij. Let us also define the Leontief inverse $\mathbf{\Gamma}\equiv\left(\mathbf{I}_{(NK)}-\mathbf{\Sigma}\right)^{-1}$, with the (nj,ik)-th entry, Γ_{ni}^{jk}, capturing the network-adjusted share of nj's revenue spent on inputs ik, either directly or indirectly through suppliers and suppliers of suppliers, ad infinitum. Finally, let $\Lambda_{ni}^j\equiv\sum_{k=1}^{K}\gamma_i^k\Gamma_{ni}^{jk}$ denote the network-adjusted input cost share of nj's revenue on value-added (labor) in country i; note that $\sum_{i=1}^{N}\Lambda_{ni}^j = 1$ for all nj due to constant returns to scale. Equation (F.81) can be re-written as:

$$\mathrm{d}\ln p_{ni}^k = \sum_{l=1}^{N}\Lambda_{il}^k\left(\mathrm{d}\ln w_l - \mathrm{d}\ln z_l\right). \tag{F.82}$$

We can now use (F.82) to re-write the linearized expenditure shares from equation (F.72) as:

$$\mathrm{d}\ln s_{ni}^{k} = \theta\left(\sum_{h=1}^{N} s_{nh}^{k}\sum_{r=1}^{N}\Lambda_{hr}^{k}\left(\mathrm{d}\ln w_r - \mathrm{d}\ln z_r\right) - \sum_{r}\Lambda_{ir}^{k}\left(\mathrm{d}\ln w_r - \mathrm{d}\ln z_r\right)\right), \tag{F.83}$$
$$= \theta\sum_{r=1}^{N}\left(\sum_{h=1}^{N} s_{nh}^{k}\Lambda_{hr}^{k} - \Lambda_{ir}^{k}\right)\left(\mathrm{d}\ln w_r - \mathrm{d}\ln z_r\right).$$

Substitute this into (F.80), we get

$$\mathrm{d}\ln w_i = \sum_{k=1}^{K}\xi_i^k\sum_{o=1}^{N}\sum_{l=1}^{K}\Delta_{io}^{kl}\sum_{n=1}^{N}\vartheta_{on}^{l}\,\mathrm{d}\ln w_n$$
$$+\sum_{k=1}^{K}\xi_i^k\sum_{o=1}^{N}\sum_{l=1}^{K}\Delta_{io}^{kl}\theta\sum_{n=1}^{N}\left(\vartheta_{on}^{l}+\sum_{j=1}^{K}\Theta_{on}^{lj}\right)\left(\sum_{r}\left(\sum_{h=1}^{N}s_{nh}^{l}\Lambda_{hr}^{l}-\Lambda_{or}^{l}\right)\left(\mathrm{d}\ln w_r - \mathrm{d}\ln z_r\right)\right).$$

To simplify notation, let us now define $\Pi_{io}^{l} \equiv \sum_{k=1}^{K}\xi_i^k\Delta_{io}^{kl}$ to be the network-adjusted share of income in country i derived from selling to country o industry l. Also denote $\Upsilon_{nor}^{l} \equiv \sum_{h=1}^{N}s_{nh}^{l}\Lambda_{hr}^{l} - \Lambda_{or}^{l}$; $\theta\Upsilon_{nor}^{l}$ is the elasticity of n's expenditure on goods lo with respect to r's factor cost. Then the above expression can be re-written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N}\underbrace{\left(\sum_{o=1}^{N}\sum_{l=1}^{K}\Pi_{io}^{l}\vartheta_{on}^{l}\right)}_{\equiv\mathbf{T}_{in}}\mathrm{d}\ln w_n$$
$$+\theta\sum_{n=1}^{N}\underbrace{\left(\sum_{r=1}^{N}\sum_{o=1}^{N}\sum_{l=1}^{K}\Pi_{io}^{l}\left(\vartheta_{or}^{l}+\sum_{j=1}^{K}\Theta_{or}^{lj}\right)\Upsilon_{ron}^{l}\right)}_{\equiv\mathbf{M}_{in}}\left(\mathrm{d}\ln w_n - \mathrm{d}\ln z_n\right) \tag{F.84}$$

Finally, we define $\mathbf{T}$ as an $N\times N$ matrix with entries $\mathbf{T}_{in} \equiv \sum_{o=1}^{N}\sum_{l=1}^{K}\Pi_{io}^{l}\vartheta_{on}^{l}$. Element $\mathbf{T}_{in}$ captures the network-adjusted share of i's income derived from selling to consumers in country n; it sums across the network-adjusted income share that country i derives from selling to country-industry ol, times the revenue share that ol derives from selling to consumers in country n. We define $\mathbf{M}$ as an $N\times N$ matrix with entries $\mathbf{M}_{in}$:

$$\mathbf{M}_{in} \equiv \sum_{r=1}^{N}\sum_{o=1}^{N}\sum_{l=1}^{K}\Pi_{io}^{l}\left(\vartheta_{or}^{l}+\sum_{j=1}^{K}\Theta_{or}^{lj}\right)\Upsilon_{ron}^{l}.$$

To interpret, $\theta\Upsilon_{ron}^{l}$ captures how r's expenditure on goods ol responds to factor cost in n. Element $\mathbf{M}_{in}$ sums the cross-substitution effects across i's exposure to all markets through network linkages: Π_{io}^{l} is i's network-adjusted income share derived from selling to producers in country o industry l; goods ol are then exposed to substitution due to changes in n's factor costs through markets that ol supplies to, including consumers (ϑ_{or}^{l}) and producers $\left(\sum_{j=1}^{K}\Theta_{or}^{lj}\right)$ in all countries (r).

We have thus obtained the same matrix representation as in the paper:

$$\mathrm{d}\ln\mathbf{w} = \mathbf{T}\,\mathrm{d}\ln\mathbf{w} + \theta\mathbf{M}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right). \tag{F.85}$$

We again solve for our friend-enemy income exposure measure by matrix inversion:

$$\mathrm{d}\ln\mathbf{w} = \mathbf{W}\,\mathrm{d}\ln\mathbf{z}, \tag{F.86}$$

where

$$\mathbf{W} \equiv -\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\mathbf{M} \tag{F.87}$$

and, using our choice of world GDP as numeraire, which implies $\mathbf{Q}\,\mathrm{d}\ln\mathbf{w}=0$,

$$\mathbf{V} \equiv \frac{\mathbf{T}+\theta\mathbf{T}\mathbf{S}}{\theta+1}-\mathbf{Q}.$$

F.6.9 Welfare

Returning to our expression for indirect utility (F.68), we have:

$$u_n = \frac{w_n}{\prod_{k=1}^{K}\left(p_n^k\right)^{\alpha_n^k}}.$$

Totally differentiating this expression for indirect utility, we have:

$$\mathrm{d}u_n = \frac{\frac{\mathrm{d}w_n}{w_n}w_n}{\prod_{k=1}^{K}\left(p_n^k\right)^{\alpha_n^k}} - \sum_{k=1}^{K}\frac{\alpha_n^k\frac{\mathrm{d}p_n^k}{p_n^k}w_n}{\prod_{k=1}^{K}\left(p_n^k\right)^{\alpha_n^k}},$$

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{k=1}^{K}\alpha_n^k\frac{\mathrm{d}p_n^k}{p_n^k}.$$

Using our total derivative of the sectoral price index above, we get:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{k=1}^{K}\alpha_n^k\sum_{m=1}^{N}s_{nm}^k\frac{\mathrm{d}p_{nm}^k}{p_{nm}^k}, \tag{F.88}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{k=1}^{K}\alpha_n^k\sum_{m=1}^{N}s_{nm}^k\,\mathrm{d}\ln p_{nm}^k.$$

Plugging (F.82) into the above, we get

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N}\underbrace{\left(\sum_{k=1}^{K}\sum_{m=1}^{N}\alpha_n^k s_{nm}^k\Lambda_{mi}^k\right)}_{\mathbf{S}_{ni}}\left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right)$$

We define $\mathbf{S}$ as an $N\times N$ matrix with entries $\mathbf{S}_{ni}\equiv\sum_{k=1}^{K}\sum_{m=1}^{N}\alpha_n^k s_{nm}^k\Lambda_{mi}^k$. Element $\mathbf{S}_{ni}$ captures the network-adjusted expenditure share of consumer n on value-added by country i; it sums across the expenditure share of consumer n on goods mk, times the network-adjusted input cost share of mk on factor i, captured by Λ_{mi}^k. We have thus obtained the same matrix representation for welfare as in the paper:

$$\mathrm{d}\ln\mathbf{u} = \mathrm{d}\ln\mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right). \tag{F.89}$$

We can re-write the above relationship as:

$$\mathrm{d}\ln\mathbf{u} = \left(\mathbf{I}-\mathbf{S}\right)\,\mathrm{d}\ln\mathbf{w} + \mathbf{S}\,\mathrm{d}\ln\mathbf{z},$$

which, using our solution for $\mathrm{d}\ln\mathbf{w}$ from equation (F.86), can be further re-written as:

$$\begin{aligned}\mathrm{d}\ln\mathbf{u} &= \left(\mathbf{I}-\mathbf{S}\right)\mathbf{W}\,\mathrm{d}\ln\mathbf{z} + \mathbf{S}\,\mathrm{d}\ln\mathbf{z},\\ &= \left[\left(\mathbf{I}-\mathbf{S}\right)\mathbf{W}+\mathbf{S}\right]\,\mathrm{d}\ln\mathbf{z},\\ &= \mathbf{U}\,\mathrm{d}\ln\mathbf{z},\end{aligned}$$

where $\mathbf{U}$ is our friend-enemy welfare exposure measure:

$$\mathbf{U} \equiv [(\mathbf{I} - \mathbf{S})\,\mathbf{W} + \mathbf{S}]\,. \tag{F.90}$$

F.6.10 Industry-Level Sales Exposure

Similarly to the multi-sector model without input linkages, our approach also yields bilateral friend-enemy measures of sales exposure to global productivity shocks for each sector. From equations (F.79) and (F.83) we obtain the following expression for changes in the sales of industry k in country i:

$$\begin{aligned} \mathrm{d}\ln y_i^k = & \sum_{n=1}^{N} \underbrace{\left(\sum_{o=1}^{N}\sum_{l=1}^{K} \Delta_{io}^{kl}\vartheta_{on}^{l}\right)}_{\equiv \mathbf{T}_{in}^{k}} \mathrm{d}\ln w_n, \\ & + \theta \sum_{n=1}^{N} \underbrace{\left(\sum_{r=1}^{N}\sum_{o=1}^{N}\sum_{l=1}^{K} \Delta_{io}^{kl}\left(\vartheta_{or}^{l} + \sum_{j=1}^{K}\Theta_{or}^{lj}\right)\Upsilon_{ron}^{l}\right)}_{\equiv \mathbf{M}_{in}^{k}} (\,\mathrm{d}\ln w_n - \,\mathrm{d}\ln z_n)\,. \end{aligned}$$

We define $\mathbf{T}^k$ as an $N \times N$ matrix with entries $\mathbf{T}_{in}^k \equiv \sum_{o=1}^{N}\sum_{l=1}^{K}\Delta_{io}^{kl}\vartheta_{on}^{l}$. Element $\mathbf{T}_{in}^k$ captures the network-adjusted share of sector ik's income derived from selling to consumers in country n; it sums across the network-adjusted income share that sector ik derives from selling to country-industry ol, times the revenue share that ol derives from selling to consumers in country n. We define $\mathbf{M}^k$ as an $N \times N$ matrix with entries $\mathbf{M}_{in}^k$:

$$\mathbf{M}_{in}^{k} \equiv \sum_{r=1}^{N}\sum_{o=1}^{N}\sum_{l=1}^{K} \Delta_{io}^{kl}\left(\vartheta_{or}^{l} + \sum_{j=1}^{K}\Theta_{or}^{lj}\right)\Upsilon_{ron}^{l}.$$

To interpret, $\theta\Upsilon_{ron}^{l}$ captures how r's expenditure on goods ol responds to factor cost in n. Element $\mathbf{M}_{in}^k$ sums the cross-substitution effects across sector ik's exposure to all markets through network linkages: Δ_{io}^{kl} is ik's network-adjusted income share derived from selling to producers in country o industry l; goods ol are then exposed to substitution due to changes in n's factor costs through markets that ol supplies to.

We get the following matrix representation:

$$\begin{aligned} \mathrm{d}\ln \mathbf{Y}^{\mathbf{k}} &= \mathbf{T}^{\mathbf{k}}\,\mathrm{d}\ln \mathbf{w} + \theta\mathbf{M}^{\mathbf{k}}\,(\,\mathrm{d}\ln \mathbf{w} - \,\mathrm{d}\ln \mathbf{z})\,, \\ &= \left[\mathbf{T}^{\mathbf{k}}\mathbf{W} + \theta\mathbf{M}^{\mathbf{k}}\,(\mathbf{W} - \mathbf{I})\right]\,\mathrm{d}\ln \mathbf{z}, \end{aligned} \tag{F.91}$$

which can be re-written as:

$$\mathrm{d}\ln \mathbf{Y}^{\mathbf{k}} = \mathbf{W}^{\mathbf{k}}\,\mathrm{d}\ln \mathbf{z}, \tag{F.92}$$

$$\mathbf{W}^{\mathbf{k}} \equiv \mathbf{T}^{\mathbf{k}}\mathbf{W} + \theta\mathbf{M}^{\mathbf{k}}\,(\mathbf{W} - \mathbf{I})\,, \tag{F.93}$$

which corresponds to our friends-and-enemies measure of sector sales exposure to productivity shocks. Note that $\mathbf{W}^{\mathbf{k}}$ also captures sector value-added exposure to productivity shocks, as value added is a constant share of revenues in this model. Finally, recall from equation (F.76) that

$$\mathrm{d}\ln w_n = \sum_{j=1}^{K} \xi_n^k\,\mathrm{d}\ln y_n^k.$$

where ξ_n^k is the value-added share of industry k in country n's total income. Together, this relationship and equations (F.86) and (F.92) imply that:

$$\mathbf{W}_i \,\mathrm{d}\ln \mathbf{z} = \sum_{k=1}^{K} \xi_n^k \mathbf{W}_i^k \,\mathrm{d}\ln \mathbf{z},$$

where $\mathbf{W}_i$ is the income exposure vector for country i with respect to productivity shocks in its trade partners and $\mathbf{W}_i^k$ is the sector value-added exposure vector for country i and sector k with respect to productivity shocks in those trade partners. It follows that our aggregate friends-and-enemies measure of income exposure ($\mathbf{W}_i$) is a weighted average of our industry friends-and-enemies measures of sector value-added exposure ($\mathbf{W}_i^k$):

$$\mathbf{W}_i = \sum_{k=1}^{K} \xi_n^k \mathbf{W}_i^k.$$

Therefore, we can decompose how much of our aggregate income exposure measure is driven by the value-added exposure of particular industries, and how much of that value-added exposure of particular industries is explained by various terms (market-size, cross-substitution, etc. within the industry).

F.6.11 Isomorphisms

Although for expositional convenience we focus on an extension of the constant elasticity Armington model with multiple sectors and input-output linkages, the same results hold in an extension of the Eaton and Kortum (2002) model to incorporate both multiple sectors following Costinot, Donaldson and Komunjer (2012) and also input-output linkages following Caliendo and Parro (2015).

F.7 Economic Geography

In this section of the online appendix, show that our approach also generalizes to models of economic geography, in which goods and factors are mobile across locations, as in Allen and Arkolakis (2014), Ramondo, Rodríguez-Clare and Saborío-Rodríguez (2016), Redding and Sturm (2008) and Redding (2016), as discussed in Section 4.6 of the paper. The world economy consists of a set of locations indexed by $i, n \in \{1, \ldots, N\}$. The economy as a whole has an exogenous supply of $\bar{\ell}$ workers, who are each endowed with one unit of labor that is supplied inelastically. Workers are perfectly mobile across locations, but have idiosyncratic preferences for each location.

F.7.1 Consumer Preferences

The preferences of worker ν who chooses to live in location n are characterized by the following indirect utility function:

$$u_n(\nu) = \frac{b_n \epsilon_n(\nu)\, w_n}{p_n}, \tag{F.94}$$

where w_n is the wage, p_n is the consumption goods price index; b_n captures amenities that are common for all workers (such as climate and scenic views); and $\epsilon_n(\nu)$ is an idiosyncratic amenity draw that is specific to each worker ν and location n. The consumption goods price index is modeled as in our baseline constant elasticity Armington model in Section 3 of the paper and takes the following form:

$$p_n = \left[\sum_{i=1}^{N} p_{ni}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1. \tag{F.95}$$

Idiosyncratic amenities are drawn independently for each worker and location from the following independent Fréchet distribution:

$$F_{\epsilon}(\epsilon) = \exp\left(-\epsilon^{-\kappa}\right), \qquad \kappa > 1, \tag{F.96}$$

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to b_n; and $\kappa > 1$ controls the dispersion of idiosyncratic amenities.

F.7.2 Production Technology

Goods are produced with labor according to a constant returns to scale production technology. These goods can be traded between locations subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from location i to location n in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Therefore, the cost to the consumer in location n of purchasing the good produced by location i is:

$$p_{ni} = \frac{\tau_{ni} w_i}{z_i}, \tag{F.97}$$

where z_i captures productivity in location i and iceberg variable trade costs satisfy $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$.

F.7.3 Expenditure Shares

Using the properties of CES demand, location n's share of expenditure on goods produced in location i is:

$$s_{ni} = \frac{\left(\tau_{ni} w_i / z_i\right)^{1-\sigma}}{\sum_{m=1}^{N} \left(\tau_{nm} w_m / z_m\right)^{1-\sigma}}. \tag{F.98}$$

Totally differentiating this expenditure share equation we get:

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = (\sigma - 1)\left(\sum_{h=1}^{N} s_{nh} \frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}}\right), \tag{F.99}$$

$$\mathrm{d}\ln s_{ni} = (\sigma - 1)\left(\sum_{h=1}^{N} s_{nh}\, \mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni}\right),$$

where from the expression for equilibrium prices (F.97) above, we have:

$$\frac{\mathrm{d}p_{ni}}{p_{ni}} = \frac{\mathrm{d}\tau_{ni}}{\tau_{ni}} + \frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i}{z_i}, \tag{F.100}$$

$$\mathrm{d}\ln p_{ni} = \mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i.$$

F.7.4 Price Indices

Totally differentiating the consumption goods price index (F.95), we have:

$$\frac{\mathrm{d}p_n}{p_n} = \sum_{m=1}^{N} s_{nm} \frac{\mathrm{d}p_{nm}}{p_{nm}}, \tag{F.101}$$

$$\mathrm{d}\ln p_n = \sum_{m=1}^{N} s_{nm}\, \mathrm{d}\ln p_{nm}.$$

F.7.5 Location Choice Probabilities

Using the properties of the Fréchet distribution (F.96), the probability that a worker chooses to live in location n is:

$$\xi_n \equiv \frac{\ell_n}{\bar{\ell}} = \frac{(b_n w_n / p_n)^{\kappa}}{\sum_{h=1}^{N} (b_h w_h / p_h)^{\kappa}}, \tag{F.102}$$

and expected utility conditional on choosing to live in a location is the same across all locations and given by:

$$\mathbb{E}[u] = \bar{u} = \Gamma\left(\frac{\kappa-1}{\kappa}\right)\left[\sum_{h=1}^{N} (b_h w_h / p_h)^{\kappa}\right]^{\frac{1}{\kappa}}. \tag{F.103}$$

Totally differentiating the location choice probabilities, we have:

$$\frac{\mathrm{d}\xi_n}{\xi_n} = \kappa\left(\frac{\mathrm{d}b_n}{b_n} + \frac{\mathrm{d}w_n}{w_n} - \frac{\mathrm{d}p_n}{p_n}\right) - \kappa \sum_{\ell=1}^{N} \xi_h \left(\frac{\mathrm{d}b_h}{b_h} + \frac{\mathrm{d}w_h}{w_h} - \frac{\mathrm{d}p_h}{p_h}\right). \tag{F.104}$$

Using the total derivative of the consumption goods price index (F.101), we can rewrite this total derivative of the location choice probabilities as:

$$\frac{d\xi_n}{\xi_n} = \kappa\left(\frac{\mathrm{d}b_n}{b_n} + \frac{\mathrm{d}w_n}{w_n} - \sum_{m=1}^{N} s_{nm}\frac{\mathrm{d}p_{nm}}{p_{nm}}\right) - \kappa \sum_{h=1}^{N} \xi_h \left(\frac{\mathrm{d}b_h}{b_h} + \frac{\mathrm{d}w_h}{w_h} - \sum_{m=1}^{N} s_{hm}\frac{\mathrm{d}p_{hm}}{p_{hm}}\right),$$

which can be further rewritten as:

$$\mathrm{d}\ln\xi_n = \left[\begin{array}{c} \kappa\left(\mathrm{d}\ln b_n + \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\,\mathrm{d}\ln p_{nm}\right) \\ -\kappa\sum_{h=1}^{N}\xi_h\left(\mathrm{d}\ln b_h + \mathrm{d}\ln w_h - \sum_{m=1}^{N} s_{hm}\,\mathrm{d}\ln p_{hm}\right) \end{array}\right]. \tag{F.105}$$

Totally differentiating expected utility, we have:

$$\frac{\mathrm{d}\bar{u}}{\bar{u}} = \sum_{h=1}^{N} \xi_h \left(\frac{\mathrm{d}b_h}{b_h} + \frac{\mathrm{d}w_h}{w_h} - \frac{\mathrm{d}p_h}{p_h}\right).$$

Using the total derivative of the consumption goods price index (F.101), we can rewrite this total derivative of expected utility as:

$$\frac{\mathrm{d}\bar{u}}{\bar{u}} = \sum_{h=1}^{N} \xi_h \left(\frac{\mathrm{d}b_h}{b_h} + \frac{\mathrm{d}w_h}{w_h} - \sum_{m=1}^{N} s_{hm}\frac{\mathrm{d}p_{hm}}{p_{hm}}\right),$$

which equivalently can be written as:

$$\mathrm{d}\ln\bar{u} = \sum_{h=1}^{N} \xi_h \left(\mathrm{d}\ln b_h + \mathrm{d}\ln w_h - \sum_{m=1}^{N} s_{hm}\,\mathrm{d}\ln p_{hm}\right). \tag{F.106}$$

F.7.6 Market Clearing

Market clearing requires that income in each location equals expenditure on goods produced in that location:

$$w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n, \tag{F.107}$$

which can be re-written as:

$$w_i \xi_i = \sum_{n=1}^{N} s_{ni} w_n \xi_n.$$

Totally differentiating this market clearing condition, we have:

$$\frac{\mathrm{d}w_i}{w_i}w_i\xi_i + \frac{\mathrm{d}\xi_i}{\xi_i}w_i\xi_i = \sum_{n=1}^{N}\frac{\mathrm{d}s_{ni}}{s_{ni}}s_{ni}w_n\xi_n + \sum_{n=1}^{N}\frac{\mathrm{d}w_n}{w_n}s_{ni}w_n\xi_n + \sum_{n=1}^{N}\frac{\mathrm{d}\xi_n}{\xi_n}s_{ni}w_n\xi_n,$$

$$\frac{\mathrm{d}w_i}{w_i}w_i\xi_i + \frac{\mathrm{d}\xi_i}{\xi_i}w_i\xi_i = \sum_{n=1}^{N}s_{ni}w_n\xi_n\left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}} + \frac{\mathrm{d}\xi_n}{\xi_n}\right).$$

Using our total derivative of expenditure shares (F.99), this becomes:

$$\frac{\mathrm{d}w_i}{w_i}w_i\xi_i + \frac{\mathrm{d}\xi_i}{\xi_i}w_i\xi_i = \sum_{n=1}^{N}s_{ni}w_n\xi_n\left(\frac{\mathrm{d}w_n}{w_n} + (\sigma-1)\left(\sum_{h=1}^{N}s_{nh}\frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}}\right) + \frac{\mathrm{d}\xi_n}{\xi_n}\right),$$

$$\frac{\mathrm{d}w_i}{w_i} + \frac{\mathrm{d}\xi_i}{\xi_i} = \sum_{n=1}^{N}\frac{s_{ni}w_n\xi_n}{w_i\xi_i}\left(\frac{\mathrm{d}w_n}{w_n} + (\sigma-1)\left(\sum_{h=1}^{N}s_{nh}\frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}}\right) + \frac{\mathrm{d}\xi_n}{\xi_n}\right),$$

$$\frac{\mathrm{d}w_i}{w_i} + \frac{\mathrm{d}\xi_i}{\xi_i} = \sum_{n=1}^{N}t_{in}\left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}\xi_n}{\xi_n} + (\sigma-1)\left(\sum_{h=1}^{N}s_{nh}\frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}}\right)\right),$$

where we have defined t_{in} as the share of location i's income from market n:

$$t_{in} \equiv \frac{s_{ni}w_nL_n}{w_iL_i},$$

and equivalently we can write this expression as:

$$\mathrm{d}\ln w_i + \mathrm{d}\ln\xi_i = \sum_{n=1}^{N}t_{in}\left(\mathrm{d}\ln w_n + \mathrm{d}\ln\xi_n + (\sigma-1)\left(\sum_{h=1}^{N}s_{nh}\,\mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni}\right)\right). \tag{F.108}$$

F.7.7 Productivity Shocks

We consider small productivity shocks, holding constant amenities and bilateral trade costs:

$$\begin{aligned} d\ln b_i &= 0, \quad \forall\, i \in N, \\ d\ln\tau_{ni} &= 0, \quad \forall\, n, i \in N, \end{aligned} \tag{F.109}$$

Using this assumption and the total derivative of prices (F.100) in the total derivative of the choice probabilities (F.105), we obtain:

$$\mathrm{d}\ln\xi_n = \kappa\left(\mathrm{d}\ln w_n - \sum_{m=1}^{N}s_{nm}\left(\mathrm{d}\ln w_m - \mathrm{d}\ln z_m\right)\right) - \kappa\sum_{h=1}^{N}\xi_h\left(\mathrm{d}\ln w_h - \sum_{m=1}^{N}s_{hm}\left(\mathrm{d}\ln w_m - \mathrm{d}\ln z_m\right)\right),$$

which can be rewritten in matrix form as:

$$\mathrm{d}\ln\boldsymbol{\xi} = \kappa\left(\mathbf{I} - \boldsymbol{\Xi}\right)\left[\mathrm{d}\ln\mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right)\right], \tag{F.110}$$

where the term inside the square parentheses is the change in the real wage change in each location and the matrix $\boldsymbol{\Xi}$ captures the *population share* of each location:

$$\boldsymbol{\Xi} = \underbrace{\begin{pmatrix} \xi_1 & \xi_2 & \cdots & \xi_N \\ \xi_1 & \xi_2 & \cdots & \xi_N \\ \vdots & \vdots & \ddots & \vdots \\ \xi_1 & \xi_2 & \dots & \xi_N \end{pmatrix}}_{N\times N}.$$

Using our assumption of no changes in amenities or bilateral trade costs in equation (F.109) and the market clearing condition (F.108), the impact of the productivity shock on income in equation (F.108) can be re-written as:

$$\mathrm{d}\ln w_i + \mathrm{d}\ln \xi_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + \mathrm{d}\ln \xi_n + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right] \right) \right),$$

which can be rewritten in matrix form as:

$$\mathrm{d}\ln \mathbf{w} + \mathrm{d}\ln \boldsymbol{\xi} = \mathbf{T} \left(\mathrm{d}\ln \mathbf{w} + \mathrm{d}\ln \boldsymbol{\xi} \right) + (\sigma - 1) \left(\mathbf{TS} - \mathbf{I} \right) \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right). \tag{F.111}$$

Substituting for the change in population share ($\mathrm{d}\ln \boldsymbol{\xi}$) using equation (F.110), we obtain:

$$\begin{bmatrix} (1+\kappa)\ \mathrm{d}\ln \mathbf{w} \\ -\kappa \mathbf{S} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right) \end{bmatrix} = \begin{bmatrix} \mathbf{T} \left[(1+\kappa)\ \mathrm{d}\ln \mathbf{w} - \kappa \mathbf{S} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right) \right] \\ + (\sigma - 1) \left(\mathbf{TS} - \mathbf{I} \right) \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right) \end{bmatrix},$$

which can be rewritten as:

$$(1+\kappa)\ \mathrm{d}\ln \mathbf{w} = (1+\kappa) \mathbf{T}\, \mathrm{d}\ln \mathbf{w} + \left[\left((\sigma - 1) - \kappa \right) \mathbf{TS} - (\sigma - 1) \mathbf{I} + \kappa \mathbf{S} \right] \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right),$$

and hence the impact of the productivity shock on wages is:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\, \mathrm{d}\ln \mathbf{w} + \left[\left(\frac{(\sigma - 1) - \kappa}{1+\kappa} \right) \mathbf{TS} - \left(\frac{\sigma - 1}{1+\kappa} \right) \mathbf{I} + \frac{\kappa}{1+\kappa} \mathbf{S} \right] \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right).$$

Using our assumed productivity shock (F.109), the total derivative of prices (F.100), and the total derivative of expected utility (F.106), the impact of the productivity shock on welfare is:

$$\mathrm{d}\ln \bar{u} = \sum_{h=1}^{N} \xi_h \left[\mathrm{d}\ln w_h - \sum_{m=1}^{N} s_{hm} \left(d\ln w_m - d\ln z_m \right) \right],$$

which can be written in matrix form as:

$$\mathrm{d}\ln \bar{u} = \xi' \left[\mathrm{d}\ln \mathbf{w} - \mathbf{S} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right) \right], \tag{F.112}$$

where the term inside the square parentheses is the change in the real wage in each location.

G Additional Empirical Results

In this section of the online appendix, we report additional empirical results, as discussed in the paper.

G.1 Economic Friends and Enemies

We begin by reporting additional empirical results for country income and welfare exposure to productivity and trade cost shocks, as discussed in Section 5 of the paper.

G.1.1 Quality of the Approximation

In this section, we report further empirical evidence on the quality of our approximation to the full non-linear solution of the model, as discussed in Section 5.2 of the paper.

Empirical Distribution of Productivity and Trade Cost Shocks To compare our linearization with exact-hat algebra for empirically-reasonable shocks, we recover the empirical distribution of productivity and trade cost shocks that rationalize the observed trade data within our baseline single-sector constant elasticity Armington model from Section D of this online appendix. Changes in trade costs and productivity are only separately identified up to a normalization or choice of units, because an increase in a country's productivity is isomorphic to a reduction in its trade costs with all partners (including itself). We use the normalization that there are no changes in own trade costs over time ($\hat{\tau}_{nnt} = 1 \; \forall n, t$), which absorbs common unobserved changes in trade costs across all partners into changes in productivity. But our results are not sensitive to the way in which we recover productivity shocks, as explored in the Monte Carlo simulations discussed in the paper and further below.

Using this normalization, we estimate time-varying bilateral trade cost shocks following Head and Ries (2001). Specifically, let x_{nit} denote the expenditure by country n on goods from country i in year t, then

$$\frac{x_{nit}}{x_{nnt}} = \frac{w_{it}^{-\theta} z_{it}^{\theta} \left(\tau_{nit}\right)^{-\theta}}{w_{nt}^{-\theta} z_{nt}^{\theta} \left(\tau_{nnt}\right)^{-\theta}},$$

which implies

$$\left(\frac{x_{nit}}{x_{nnt}} \frac{x_{int}}{x_{iit}}\right)^{\frac{1}{2}} = \left(\frac{\tau_{nit}}{\tau_{nnt}} \frac{\tau_{int}}{\tau_{iit}}\right)^{-\frac{\theta}{2}}.$$

Denoting by $\hat{x}$ relative changes in variable x across periods and using our normalization that $\hat{\tau}_{nnt} = \hat{\tau}_{iit} = 1$,

$$\left(\frac{\hat{x}_{nit}}{\hat{x}_{nnt}} \frac{\hat{x}_{int}}{\hat{x}_{iit}}\right)^{\frac{1}{2}} = \left(\hat{\tau}_{nit} \hat{\tau}_{int}\right)^{-\frac{\theta}{2}}.$$

Assuming a standard value for the trade elasticity of $\theta = 5$ and that bilateral trade cost shocks are symmetric ($\hat{\tau}_{nit} = \hat{\tau}_{int}$), we can recover all bilateral relative changes in trade costs from the bilateral trade data.

Using the market clearing condition that equates a country's income with expenditure on its goods, and again assuming a standard value for the trade elasticity of $\theta = 5$, we estimate the changes in productivity in each country ($\hat{z}_{it}$) that exactly rationalize the observed changes in per capita incomes ($\hat{w}_{it}$) and populations ($\hat{\ell}_{it}$), given our estimated changes in bilateral trade costs ($\hat{\tau}_{nit}^{-\theta}$):

$$\hat{w}_{it} \hat{\ell}_{it} w_{it} \ell_{it} = \sum_{n=1}^{N} \frac{s_{nit} \hat{\tau}_{nit}^{-\theta} \hat{w}_{it}^{-\theta} \hat{z}_{it}^{\theta}}{\sum_{\ell=1}^{N} s_{n\ell t} \hat{\tau}_{n\ell t}^{-\theta} \hat{w}_{\ell t}^{-\theta} \hat{z}_{\ell t}^{\theta}} \hat{w}_{nt} \hat{\ell}_{nt} w_{nt} \ell_{nt}, \tag{G.1}$$

where any omitted changes in trade costs for an importer n that are common across exporters cancel from the numerator and denominator of the fraction on the right-hand side; any omitted changes in trade costs for an exporter i that are common across importers are implicitly absorbed into the estimated changes in productivities ($\hat{z}_{it}$). Note that the fraction on the right-hand side of equation (G.1) corresponds to an expenditure share and is homogenous of degree zero in these changes in productivities ($\hat{z}_{it}$). Therefore, these changes in productivities ($\hat{z}_{it}$) only can be recovered up to a normalization or choice of units, and we use the normalization that the mean of the log changes in productivities across all countries is equal to zero (a geometric mean of changes in productivities of one).

Having recovered these changes in productivities ($\hat{z}_{it}$) implied by the observed data, we compare the predictions from our (first-order) linearization for the impact of productivity shocks on income to those from the full non-linear solution of the model using the exact-hat algebra approach. In particular, we undertake exact-hat algebra counterfactuals, in which we solve for the counterfactual change in per capita income ($\hat{w}_{it}$) in response to the changes in productivity alone ($\hat{z}_{it}$), holding trade costs and populations constant:

$$\hat{w}_{it} w_{it} \ell_{it} = \sum_{n=1}^{N} \frac{s_{nit} \hat{w}_{it}^{-\theta} \hat{z}_{it}^{\theta}}{\sum_{\ell=1}^{N} s_{n\ell t} \hat{w}_{\ell t}^{-\theta} \hat{z}_{\ell t}^{\theta}} \hat{w}_{nt} w_{nt} \ell_{nt}. \tag{G.2}$$

We compare the results from these exact-hat algebra counterfactuals to the predictions of our linearization, which implies a log change in countries' per capita incomes in response to these productivity shocks of $\ln \hat{\mathbf{w}} = \mathbf{W} \ln \hat{\mathbf{z}}$. We also undertake an analogous exercise in which we compare the predictions of our linearization for changes in bilateral trade costs to the counterfactual predictions from exact-hat algebra, as discussed further below.

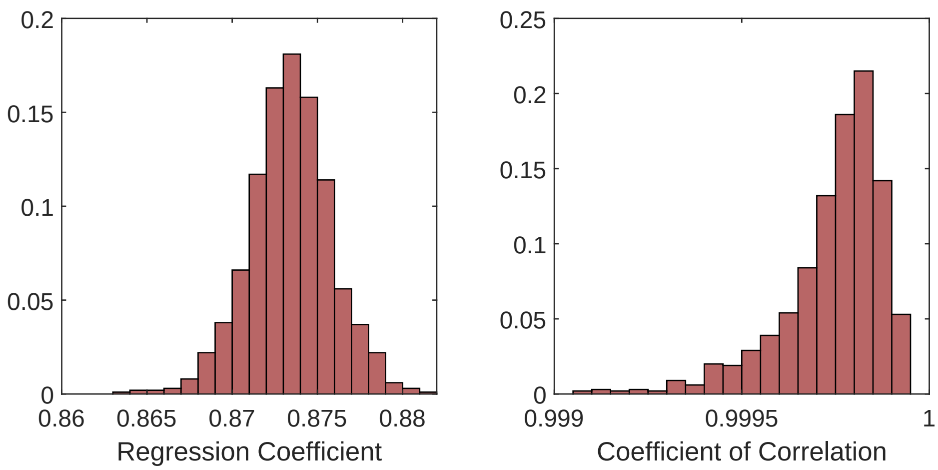
Quality of the Approximation for Productivity Shocks In Section 5.2 of the paper, we report Monte Carlo simulations for our baseline trade elasticity of $\theta = 5$, in which we draw (with replacement) productivity shocks for each country from the empirical distribution of productivity shocks from 2000-2010. Using these simulated productivity shocks, we undertake exact-hat algebra counterfactuals to compute predicted log changes in per capita income, and compare these predictions with those from our linearization. In Figure 3 in the paper, we show that we find slope coefficients from 0.99-1.01 and a correlation coefficient of above 0.999. In Figures G.1 and G.2 below, we explore the robustness of these results to alternative parameter values, by considering trade elasticities of $\theta = 2$ and $\theta = 20$, which spans the range of empirically plausible values for this parameter. Even for trade elasticities as extreme as 2 and 20, we find regressions slope coefficients ranging from 0.85-1.09 and correlation coefficients of above 0.999.

Proposition 5 in Subsection 3.6 of the paper shows that the second-order approximation error for the income exposure of each country is bounded by the product between the largest eigenvalue of the corresponding Hessian matrix and the cross-country variance of productivity shocks (i.e., $|\epsilon_i(\tilde{\mathbf{z}})| \leq |\mu^{\max,i}| \cdot \tilde{\mathbf{z}}'\tilde{\mathbf{z}}$). In Table G.1, we show the eigenvalues of the Hessian matrix. Both for the year 2000 and averaged across our sample period, the largest eigenvalue is small, implying that the second-order approximation error for the income exposure of each country is bounded by 0.26 percent and 0.36 percent of the variance of productivity shocks. Moreover, Table G.1 shows that all other eigenvalues are substantially smaller; the second largest eigenvalue $\mu^{2nd,i}$, for instance, averages to 0.0016 across countries i for the year 2000 and to 0.0020 for the entire sample. We find that for each country i, $\tilde{\mathbf{z}}^{\max,i} \approx \mathbf{e}^i$, implying that productivity shocks represented by the standard unit vector $\mathbf{e}^i$ come close to achieving the upper bound of the approximation error, and any productivity shock vector that is orthogonal to $\tilde{\mathbf{z}}^{\max,i}$ induces approximation errors that are bounded above by $\mu^{2nd,i}$. As a result, across our entire sample, we find that the second-order approximation errors in income exposure to own-productivity shocks are, on average, bounded by 0.36 percent of the variance of productivity shocks, and the second-order approximation errors in income exposure to other countries' productivity shocks are, on average, bounded by 0.20 percent of the variance of productivity shocks.

Proposition 7 in Subsection 3.6 of the paper further shows that the eigenvalues of the Hessian matrix, evaluated at some productivity shock vector $\mathbf{x}$, provides the *exact* error for our first-order approximation, summing across second- and all higher-order errors (i.e., $\tilde{w}_i - [\mathbf{W}\tilde{\mathbf{z}}]_i = \tilde{\mathbf{z}}'\mathbf{H}_{f_i}(\mathbf{x})\tilde{\mathbf{z}}$ for some $\mathbf{x}$). A bound on the eigenvalue of the Hessian evaluated over the entire support $\mathcal{X}$ of productivity shocks therefore provides an upper-bound on the exact approximation error. We cannot exhaustively evaluate the eigenvalues of Hessians over the entire support of all possible productivity shocks, because the support of the distribution of productivity shocks is not observed in any finite sample. Nevertheless, the upper envelope of all historical realizations of the Hessian eigenvalues is informative, because year-to-year variations in the Hessian matrices are exactly due to differences in the realizations of productivity shocks over time. As reported in Table G.1, the largest eigenvalue of the Hessian, averaged across countries and taking the maximum across our sample period, is 0.0062, and subsequent eigenvalues are again substantially smaller. This implies global errors in our first-order approximation of income exposures to own productivity shocks of 0.62 percent of the variance of productivity shocks, and global approximation errors of income exposure to other countries' productivity shocks of 0.33 percent of the variance of productivity shocks.

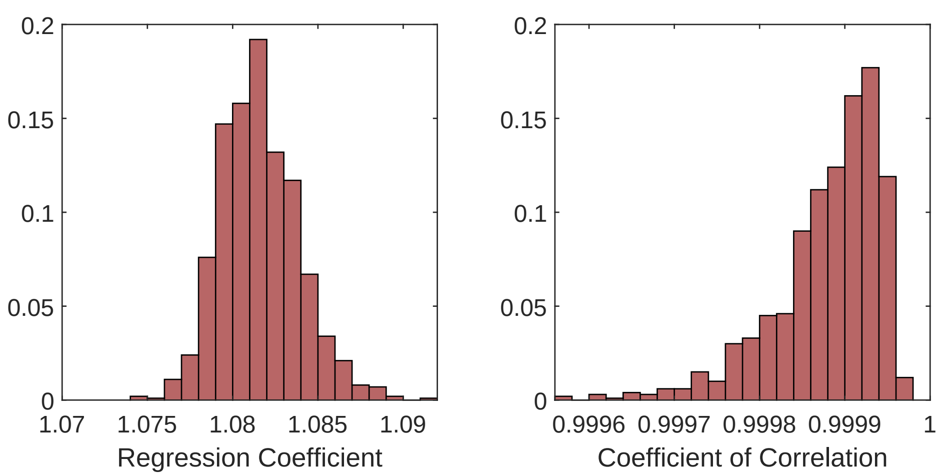
Taken together, these results suggest that our friend-enemy income exposure measures for productivity shocks are close to exact for empirically-reasonable changes in productivities and trade elasticities.

Figure G.1: Distributions of Regression Slope Coefficients and Coefficients of Correlation Comparing our Friend-Enemy Approximation to Exact-Hat Algebra Predictions in Monte Carlos using Simulated Productivity Shocks ($\theta = 2$)



Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3 of the paper. Monte Carlo simulations using 1,000 replications. Simulated productivity shocks drawn (with replacement) from the empirical distribution of productivity shocks from 2000-10.

Figure G.2: Distributions of Regression Slope Coefficients and Coefficients of Correlation Comparing our Friend-Enemy Approximation to Exact-Hat Algebra Predictions in Monte Carlos using Simulated Productivity Shocks ($\theta = 20$)



Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3 of the paper. Monte Carlo simulations using 1,000 replications. Simulated productivity shocks drawn (with replacement) from the empirical distribution of productivity shocks from 2000-10.

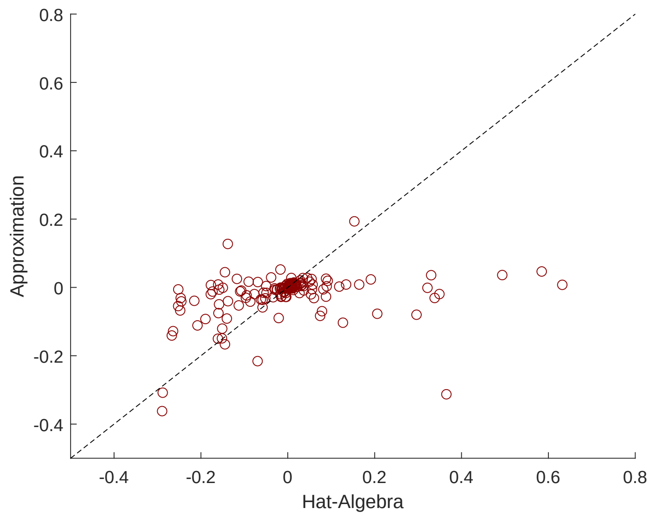
Table G.1: Eigenvalues of the Hessian Matrix

	Eigenvalues of Hessians, ordered by absolute value, averaged across all countries									
	1	2	3	4	5	6	7	8	9	10
Year 2000										
	0.0026	0.0016	0.0010	0.0008	0.0006	0.0005	0.0004	0.0003	0.0003	0.0002
Average across years 1970–2012										
	0.0036	0.0020	0.0014	0.0010	0.0008	0.0006	0.0005	0.0004	0.0004	0.0003
Max across years 1970–2012										
	0.0062	0.0033	0.0023	0.0016	0.0013	0.0010	0.0008	0.0007	0.0006	0.0005

Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3 of the paper.

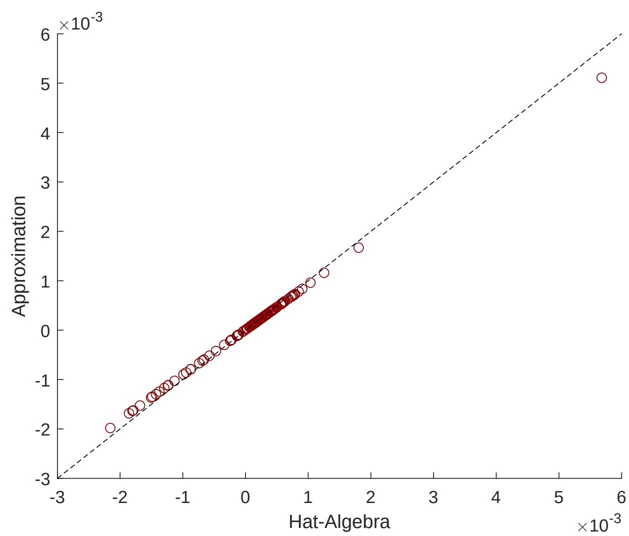
Quality of the Approximation for Trade Cost Shocks In Figure 1b in the paper, we report a comparison of predicted changes in per capita income using our linearization and exact-hat algebra counterfactuals for productivity shocks. In Figure G.3 below, we report an analogous comparison of our linearization and exact-hat algebra counterfactuals using the empirical distribution of trade cost shocks from 2000-2010. While we again find a strong relationship between the predictions of our linearization and the exact-hat algebra counterfactuals, it is noticeably weaker than for productivity shocks, with a regression slope of less than one. An important reason for this difference is that the productivity shock is common across all trade partners, which means that the direct effect of this productivity shock can be taken outside of the summation across trade partners into a separate first term that is the same in our linearization and the exact hat algebra in equations (26) and (27) in the paper. In contrast, the direct effect of bilateral changes in trade costs cannot be taken outside of this summation sign, because it varies across trade partners. Nevertheless, even though the regression slope now differs from one for trade cost shocks, the correlation coefficient between the predicted changes in per capita income using our linearization and exact-hat algebra counterfactuals remains greater than 0.99. In Monte Carlo simulations in which we draw a shock to bilateral trade costs for an exporter-importer pair from the empirical distribution of trade cost shocks from 2000-2010, and compare our linearization and the exact-hat algebra counterfactuals, we find this same pattern of results with regressions slopes that can differ from one, but correlation coefficients that remain close to one. Furthermore, although our linearization works less well in general for bilateral trade cost shocks, we find many examples in which it provides a close approximation to the full non-linear solution of the model, as shown for a shock to US-China bilateral trade costs in Figure G.4.

Figure G.3: Predicted Impact of Bilateral Trade Cost Shocks on Income: Our Friend-Enemy Approximation Versus Exact-Hat Algebra Predictions for the Empirical Distribution of Trade Cost Shocks from 2000-2010 ($\theta = 5$)



Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3 of the paper.

Figure G.4: Predicted Impact of Bilateral Trade Cost Shocks on Income: Our Friend-Enemy Approximation Versus Exact-Hat Algebra Predictions for a 50 Percent Increase in US-China Bilateral Trade Costs ($\theta = 5$)



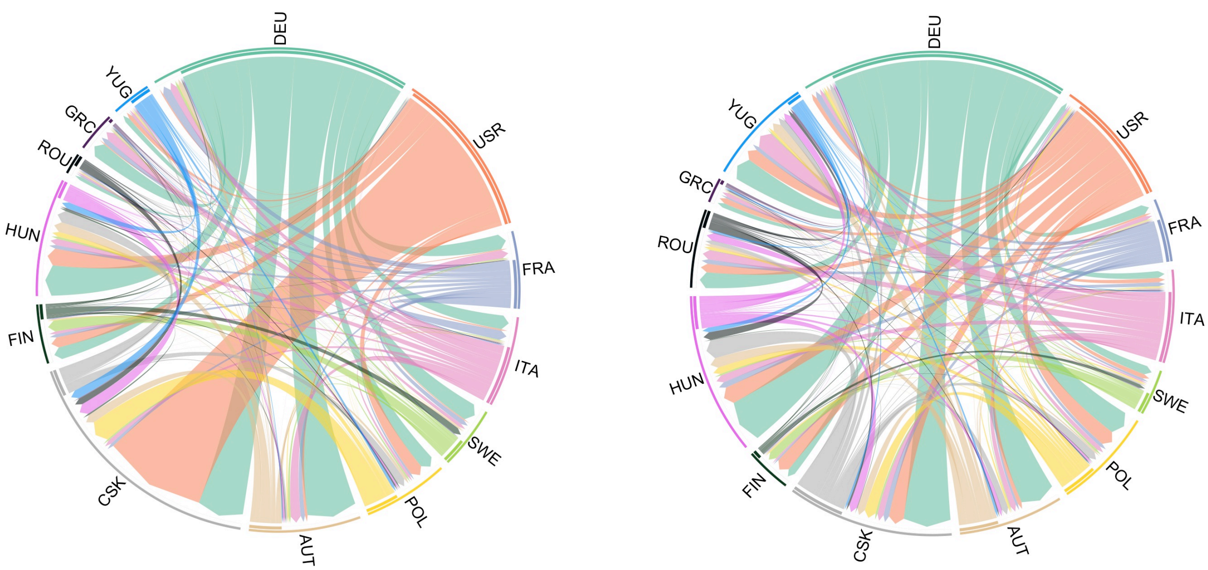
Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3 of the paper.

G.1.2 Regional Networks of Welfare Exposure

In this section, we report further empirical evidence on regional networks of welfare exposure, as discussed in Section 5.3.2 of the paper. In Figure G.5, we show bilateral welfare exposure in Central Europe before and after the fall of the Iron Curtain. In 1988 immediately before this event, we observe strong connections between the countries of the former Soviet Union (USR) and Eastern European nations such as the former Czechoslovakia (CSK). By 2012, these connections have substantially weakened, and we observe growing connections between Western European countries such as Italy and Eastern European nations. Although Germany is here the aggregation of the former and East and West Germanies in all years, we also observe a strengthening of its position at the center of the network of welfare exposure. More broadly, we also find an increase in the overall density of connections over time, consistent with trade liberalization increasing countries' economic interdependence.

Figure G.5: Central European Welfare Exposure, 1988 and 2012

(a) 1988 (b) 2012



Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3 of the paper.

G.2 Economic and Political Friends and Enemies

In this section of the online appendix, we report additional information for our analysis of the relationship between bilateral political attitudes and our friends-and-enemies sufficient statistics in Section 6 of the paper. In Subsection G.2.1, we report additional details about the measurement of bilateral political attitudes using voting similarity data. In Subsection G.2.2, we report additional empirical results for the relationship between bilateral strategic rivalries and our our friends-and-enemies sufficient statistics.

G.2.1 Measuring Political Attitudes Using Bilateral Voting Similarity

We follow a large literature in political science in measuring countries' bilateral political attitudes towards one another using the similarity of their votes in the United Nations (UN). The ultimate source for our UN voting data is Voeten (2013), which reports non-unanimous plenary votes in the United Nations General Assembly (UNGA) from 1962-2012, and includes on average around 128 votes each year. Countries are recorded as either voting "no" (coded 1), "abstain" (coded 2) or "yes" (coded 3). In particular, we use the bilateral measures of voting similarity constructed using these data in the Chance-Corrected Measures of Foreign Policy Similarity (FPSIM) database, as reported in Häge (2017). We denote the outcome of vote v for country i by $O_i(v)$:

$$O_i(v) \in \{1, 2, 3\} \qquad v \in \{1, \dots, V\}. \tag{G.3}$$

Building on a large literature in international relations, we consider a number of different measures of bilateral voting similarity. Our first and simplest measure is the S-score of Signorino and Ritter (1999), which measures the extent of agreement between the votes of countries n and i as one minus the sum of the squared actual deviation between their votes scaled by the sum of the squared maximum possible deviations between their votes:

$$\mathbb{S}_{ni}^{S} = 1 - \frac{\sum_{v=1}^{V} \left(O_n(v) - O_i(v)\right)^2}{\frac{1}{2}\sum_{v=1}^{V} \left(d_{\max}(v)\right)^2} \tag{G.4}$$

where $(d_{\max}(v))^2 = (\sup\{O_n(v) - O_i(v)\})^2$ represents the maximum possible disagreement for each vote and this measure is bounded between minus one (maximum possible disagreement) and one (maximum possible agreement).

A limitation of this S-score measure is that is does not control for properties of the empirical distribution function of country votes. In particular, country votes may align by chance, such that the frequency with which countries agree on a "yes" depends on the frequency with which countries vote "yes." Similarly, the frequency with which they agree on each of the other voting outcomes ("no" and "abstain") depends on the frequencies with they choose these other voting outcomes. Therefore, we also consider two alternative measures of countries' bilateral similarity in voting patterns that control in different ways for properties of the empirical distribution of voting outcomes. First, the π-score of Scott (1955) adjusts the observed variability of the countries' bilateral voting outcomes with the variability of each country's own voting outcomes around average outcomes across the two countries taken together:

$$\mathbb{S}_{ni}^{\pi} = 1 - \frac{\sum_{v=1}^{V} \left(O_n(v) - O_i(v)\right)^2}{\sum_{n=1}^{V} \left(O_n(v) - \frac{\bar{O}_n + \bar{O}_i}{2}\right)^2 + \sum_{v=1}^{V} \left(O_i(v) - \frac{\bar{O}_n + \bar{O}_i}{2}\right)^2}, \tag{G.5}$$

where $\bar{O}_i = (1/V)\sum_{v=1}^{V} O_i(v)$ is the average outcome for country i.

Second, the κ-score of Cohen (1960) adjusts the observed variability of the countries' bilateral voting outcomes with the variability of each country's own voting outcomes around its own average outcome and the difference between the two countries' average outcomes:

$$\mathbb{S}_{ni}^{\kappa} = 1 - \frac{\sum_{v=1}^{V} \left(O_n(v) - O_i(v)\right)^2}{\sum_{v=1}^{V} \left(O_n(v) - \bar{O}_n\right)^2 + \sum_{v=1}^{V} \left(O_i(v) - \bar{O}_i\right)^2 + \sum_{v=1}^{V} \left(\bar{O}_n - \bar{O}_i\right)^2}. \tag{G.6}$$

Both the π-score and κ-score have an attractive statistical interpretation, as discussed further in Krippendorf (1970), Fay (2005) and Häge (2011). In the case of binary (0,1) voting outcomes, these indices reduce to the form of $1 - (D_o/D_e)$, where D_o is the observed frequency of agreement and D_e is the expected frequency of agreement. The key difference between the two indices is in their assumptions about the expected frequency of agreement. The π-score estimates the expected frequency of agreement using the average of the two countries marginal distributions of voting outcomes. In contrast, the κ-score estimates the expected frequency of agreement using each country's own individual marginal distribution of voting outcomes. Whereas our economic measures of exposure are potentially asymmetric, such that n's exposure to i is not necessarily the same as i's exposure to n, each of these measures of foreign policy similarity is necessarily symmetric.

G.2.2 Additional Empirical Results on Political Attitudes

In Section 6.2 of the paper, we examine the relationship between countries' bilateral political attitudes towards one another and our friends-and-enemies sufficient statistics. We report estimation results using both measures of voting similarity in the United Nations General Assembly (UNGA) and measures of strategic rivalries. In this section of the online appendix, we report additional empirical results for strategic rivalries, using the measures of different types of strategic rivalries (positional, spatial and ideological) from Colaresi et al. (2010). Table G.2 presents the results of estimating the same instrumental variables specification as in Columns (9)-(10) in Table 1 in the paper, but using these three types of strategic rivalries. Columns (1)-(2) use positional rivalry; Columns (3)-(4) use spatial rivalry; and Columns (5)-(6) use ideological rivalry. In each of these pairs of specifications, the first column ((1), (3) and (5)) includes only the exporter-importer and year fixed effects; the second column ((2), (4) and (6)) augments this specification with exporter-year and importer-year fixed effects.

Regardless of which measure of strategic rivalry and econometric specification we consider, we find a negative estimated coefficient that is statistically significant at conventional critical values. Therefore, as countries become greater economic friends in terms of the welfare effects of their productivity growth, they become greater political friends in terms of having a lower propensity to be strategic rivals of each type (positional, spatial and ideological). Beneath the coefficient and standard error for each regression specification, we report the first-stage F-statistic. The first-stage F statistics in each panel are again the same across Columns (1), (3) and (5) and Columns (2), (4) and (6), because the first-stage regression specification (welfare exposure on the instrument) and sample size is the same across each of the different measures of strategic rivalry used in the second-stage regression. Although these first-stage F-statistics naturally fall in the specifications including importer-year and exporter-year fixed effects, they remain well above the conventional threshold of 10.

Table G.2: Bilateral Political Attitudes and Welfare Exposure (Positional, Spatial and Ideological Strategic Rivalries)

Political Outcome	Positional Rivalry		Spatial Rivalry		Ideological Rivalry	
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Welfare exposure in single-sector model						
$\mathbf{U}^{Single-sector}$	-2.932**	-3.162**	-3.131**	-3.393**	-1.841**	-1.450*
	(1.442)	(1.541)	(1.566)	(1.671)	(0.726)	(0.755)
First-stage F	99.50	78.52	99.50	78.52	99.50	78.52
Panel B: Welfare exposure in multi-sector model						
$\mathbf{U}^{Multi-sector}$	-2.904**	-3.133**	-3.100**	-3.362**	-1.823**	-1.437*
	(1.429)	(1.528)	(1.552)	(1.657)	(0.719)	(0.748)
First-stage F	99.36	78.20	99.36	78.20	99.36	78.20
Panel C: Welfare exposure in input-output model						
$\mathbf{U}^{Input-Output}$	-4.742**	-5.176**	-5.063**	-5.555**	-2.977***	-2.374**
	(2.278)	(2.455)	(2.471)	(2.655)	(1.140)	(1.211)
First-stage F	173.6	119.8	173.6	119.8	173.6	119.8
Specification: 2SLS						
Exp × Imp	Yes	Yes	Yes	Yes	Yes	Yes
Year	Yes	No	Yes	No	Yes	No
Exp × Year	No	Yes	No	Yes	No	Yes
Imp × Year	No	Yes	No	Yes	No	Yes
No. of Obs.	610954	610954	610954	610954	610954	610954
No. of Clusters	14761	14761	14761	14761	14761	14761

Standard errors in parentheses are clustered at country-pair level
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

H Data Appendix

Our data on international trade are from the NBER World Trade Database, which reports values of bilateral trade between countries for around 1,500 4-digit Standard International Trade Classification (SITC) codes. The ultimate source for these data is the United Nations COMTRADE database and we use an updated version of the dataset from Feenstra et al. (2005) for the time period 1970-2012.[1] We augment these trade data with information on countries' gross domestic product (GDP), population and bilateral distances from the GEODIST and GRAVITY datasets from CEPII.[2] Note that the NBER World Trade Database lacks direct data on a country's expenditure on domestic goods (X_{nnt}). Therefore, we compute this domestic expenditure as gross output minus exports plus imports. To measure gross output for each country, we multiply its GDP (value added) by 2.2, which is the mean ratio of gross output to GDP in the EU-KLEMS database (which includes the USA and Japan). This method yields a median self-trade-share (expenditure on own goods relative to total expenditure) of 0.92 in 1970 and of 0.83 in 2012, reflecting the increase in global trade over this period. In our multi-sector models, we report results aggregating the products in the NBER World Trade Database to the 20 International Standard Industrial Classification (ISIC) industries listed below. In specifications incorporating input-output linkages, we use a common input-output matrix for all countries from Caliendo and Parro (2015).

Table H.1: International Standard Industrial Classification (ISIC) Industries

Industry Code	Short Name	Long Description
1	Agriculture	Agriculture Agriculture forestry and Fishing
2	Mining	Mining and quarrying
3	Food	Food products, beverages and tobacco
4	Textile	Textiles, textile products, leather and footwear
5	Wood	Wood and products of wood and cork
6	Paper	Pulp, paper, paper products, printing and publishing
7	Petroleum	Coke, refined petroleum and nuclear fuel
8	Chemicals	Chemicals
9	Plastic	Rubber and plastics products
10	Minerals	Other nonmetallic mineral products
11	Basic Metals	Basic metals
12	Metal Products	Fabricated metal products, except machinery and equipment
13	Machinery nec	Machinery and equipment n.e.c
14	Office	Office, accounting and computing machinery
15	Electrical	Electrical machinery and apparatus, n.e.c.
16	Communication	Radio, television and communication equipment
17	Medical	Medical, precision and optical instruments, watches and clocks
18	Auto	Motor vehicles trailers and semi-trailers
19	Other Transport	Other transport equipment
20	Other	Manufacturing n.e.c and recycling

[1] See https://cid.econ.ucdavis.edu/wix.html.
[2] See http://www.cepii.fr/cepii/en/bdd_modele/bdd.asp.

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